Combinatorics Homework Booklet

1. When you play lotto 5-30, you must choose 5 different integers from 1 to 30. How many combinations are possible?

   A. $\frac{30!}{5!25!}$ *B.* $\frac{30!}{25!}$ *C.* $25!$ *D.* $\frac{30!}{5!}$

2. Determine the 4th term of $(x-2)^6$

   A. $120x^2$ *B.* $240x^2$ *C.* $-160x^3$ *D.* $-320x^3$

3. Determine the number of different arrangements of all the letters in APPLEPIE.

   A. 3360 *B.* 6720 *C.* 40312 *D.* 40320

4. Assume a car license plate consists of 7 characters. The first 3 characters can be any of the letters for A to F, but no letter can be repeated. The next 3 characters can be any of the digits from 1 to 9, but no digit can be repeated. The last character can be any of the letters X, Y, Z. An example of this format is: BFA648Y. How many license plates are possible?

   A. 5040 *B.* 181440 *C.* 472392 *D.* 4084080

5. Suppose you play a game of cards in which only three cards are dealt from a standard 52-card deck. How many ways are there to obtain one pair? (2 cards of the same rank and 1 card of a different rank.)

   A. 1014 *B.* 1872 *C.* 3744 *D.* 3900

6. A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.

   A. $\frac{10!}{7!}$ *B.* $\frac{10!}{3!}$ *C.* $\frac{10!}{3!7!}$ *D.* $\frac{10!}{3!3!4!}$

7. Determine the 4th term in the expansion of $(x-2y)^5$

   A. $-80x^2y^3$ *B.* $-40x^3y^2$ *C.* $40x^3y^2$ *D.* $80x^2y^3$

8. Express ${}_{33}C_5$ using factorial notation.

   A. $\frac{33!}{5!}$ *B.* $\frac{33!}{28!}$ *C.* $\frac{33!}{5!28!}$ *D.* $28!$

9. Determine the 3rd term in the expansion of $(x-y)^{10}$

   A. $-45x^8y^2$ *B.* $-120x^7y^3$ *C.* $45x^8y^2$ *D.* $120x^7y^3$

10. How many different pasta meals can be made from 4 choices of pasta and 2 choices of sauces, if only one pasta and one sauce is selected for each meal?

A. 4 *B.* 6 *C.* 8 *D.* 16

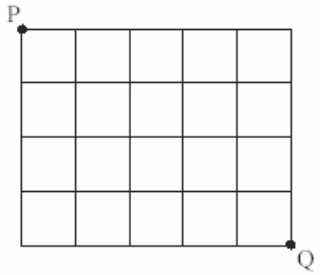
11. A man has 7 different pets and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

A. 21 *B.* 35 *C.* 210 *D.* 840

12. Determine the 3rd term of $(2x+y)^6$

A. $15x^4y^2$ *B.* $240x^4y^2$ *C.* $120x^3y^3$ *D.* $160x^3y^3$

13. Moving only to the right and down, how many different paths exist to get from point P to point Q?

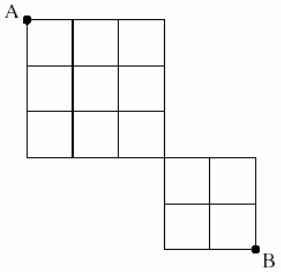


A. 120 *B.* 126 *C.* 180 *D.* 480

14. Which expression is equivalent to ${}_nC_2$?

A. n^2-2n *B.* n^2-n *C.* $\frac{1}{2}(n^2-2n)$ *D.* $\frac{1}{2}(n^2-n)$

15. Moving only to the right and down, how many different paths are there from A to B?



A. 26 *B.* 52 *C.* 120 *D.* 252

16. Simplify: $\frac{n(n+1)!}{(n-1)!}$

A. $2n!$ *B.* $n!(n^2+n)$ *C.* $2n$ *D.* n^3+n^2

17. In the expansion of $(2a-3b)^6$, determine the coefficient of the term containing a^4b^2.

A. -4320 *B.* 864 *C.* 2160 *D.* 2880

18. When playing the 6/49 lottery, a customer must choose 6 different number from 1 to 49 inclusive. How many combinations are possible?

A. 49! *B.* $\frac{49!}{6!43!}$ *C.* $\frac{49!}{43!}$ *D.* $\frac{49!}{6!}$

19. Twelve buttons differ only by colour. There are 4 red buttons, 4 green buttons and 4 yellow buttons. If the buttons are placed in a row, how many different arrangements are possible?

A. 11880 *B.* 34650 *C.* 19958400 *D.* 479001600

20. How many odd 3-digit whole numbers are there? For example, 203 is acceptable but 023 is not.

A. 360 *B.* 450 *C.* 500 *D.* 900

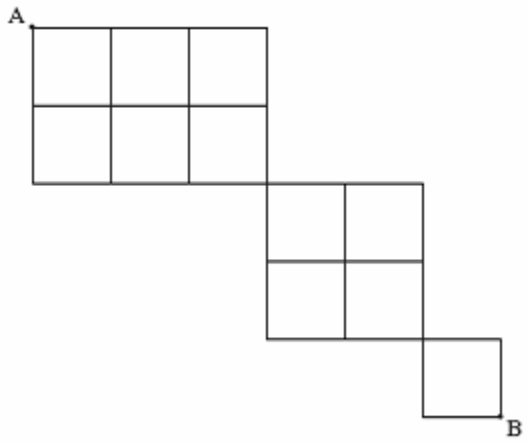
21. How many different committees of 2 people can be selected from 5 people?

A. $\frac{5!}{2!}$ *B.* $\frac{5!}{3!}$ *C.* $\frac{5!}{2!3!}$ *D.* 5!

22. Determine the 5th term in the expansion of $\left(x-\frac{1}{2}y\right)^7$

A. $\frac{35}{8}x^4y^3$ *B.* $\frac{35}{16}x^3y^4$ *C.* $-\frac{35}{8}x^4y^3$ *D.* $-\frac{35}{16}x^3y^4$

23. Moving only to the right or down, how many different paths exist to get from point A to B?



A. 22
B. 60
C. 120
D. 144

Written Questions

1. Solve algebraically: $\frac{(n-1)!}{(n-3)!}=30$

2. A class has 30 students:
   a) How many ways can a committee of 3 people be selected from the class?

   b) How many ways can an executive committee of 3 people (president, vice-president, and secretary) be selected from the class?

   c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected from the class if the committee must contain 1 boy and 2 girls?

3. A toy box contains 4 different cars and 6 different trucks.
   a) In how many ways can a collection of 5 toys be chosen if the collection must consist of 2 cars and 3 trucks?

   b) In how many ways can a collection of 5 toys be chosen if the collection must consist of at least 3 cars?

4. Determine the first three terms of the expansion: $(x-2y)^7$

5. Solve Algebraically: $\frac{n!}{(n-2)!4!}=10$

Key

1. A	2. C	3. A	4. B	5. C	6. A	7. A	8. C
9. C	10. C	11. C	12. B	13. B	14. D	15. C	16. D
17. C	18. B	19. B	20. B	21. C	22. B	23. C	

1.

$$\frac{(n-1)(n-2)(n-3)!}{(n-3)!} = 30$$

$$(n-1)(n-2) = 30$$

$$n^2 - 3n + 2 = 30$$

$$n^2 - 3n - 28 = 0$$

$$(n-7)(n+4) = 0$$

$$n = 7 \text{ or } -4 (\textit{reject})$$

$$n = 7$$

2a) 4060 ways 2b) 24360 ways 2c) 1900 ways

3a) 120 ways 3b) 66 ways

4.

$$x^7 - 14x^6y + 84x^5y^2$$

5. n = 16