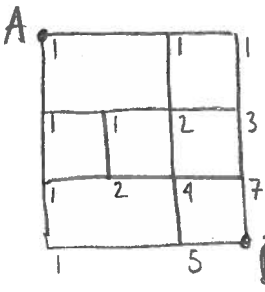
Combinatorics Review Package

C ① $(2x+y)^{9}$ → $9+1 = 10$ terms

C ② $\dfrac{(n-2)!}{(n-1)!} = \dfrac{(n-2)!}{(n-1)(n-2)!} = \dfrac{1}{n-1}$

B ③



12 ways

A ④ $\dfrac{24}{L} \cdot \dfrac{23}{L} \cdot \dfrac{22}{L} \cdot \dfrac{8}{\#} \cdot \dfrac{7}{\#} \cdot \dfrac{6}{\#} = 4\,080\,384$

A + Z less I + 0 ⟹ 24 letters (no repeats)

digits 2 to 9 ⟹ 8 numbers (no repeats)

B ⑤ $(x+2y)^{10}$ $\quad \sum_{r=0}^{2} {}_{10}C_r (x)^{10-r}(2y)^{r} = {}_{10}C_0\, x^{10}(2y)^{0} + {}_{10}C_1\, x^{9}(2y)^{1} + {}_{10}C_2\, x^{8}(2y)^{2}$

$n = 10$

$a = x$

$b = 2y$

$= (1)x^{10}(1) + (10)x^{9}(2y) + (45)x^{8}(2)^{2}(y)^{2}$

$= x^{10} + 20x^{9}y + 180x^{8}y^{2}$

C ⑥ 12 boys, 10 girls } $n = 22$ select 3 people ⟹ $r = 3$

How many committees of 3 have @ least 1 boy?

All possible committees: $22C_3 = 1540$

All girls (no boys): $10C_3 = 120$

⟹ subtract for @ least 1 boy: 1420

D ⑦ $\dfrac{4}{\text{rest}} \cdot \dfrac{6}{\text{nose}} \cdot \dfrac{4}{\text{coffee}} = 96$

⑧ TSAWWASSEN

B

$n = 10$
$S = 3$
$A = 2$
$W = 2$

$$\frac{10!}{3!\,2!\,2!} = 151\,200$$

⑨

B

X is in the 6th row, which means it's for $n = 5$ (draw out Pascal's Triangle if you'd like). And it's the third in, so $r = 2$ (begin $r = 0, 1, 2, 3, 4, 5$)

$\therefore {}_nC_r = {}_5C_2$

⑩ NANAIMO

B

$n = 7$
$N = 2$
$A = 2$

$$\frac{7!}{2!\,2!} = 1260$$

⑪

D

${}_nP_r$ w $n = 7$, $r = 4$ since order counts.

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!}$$

⑫

A

8th term $\Rightarrow r = 7$

$n = 11$ for $(2x - y)^{11}$

$a = 2x$
$b = -y$

$t_{r+1} = {}_nC_r\,(a)^{n-r}(b)^r$

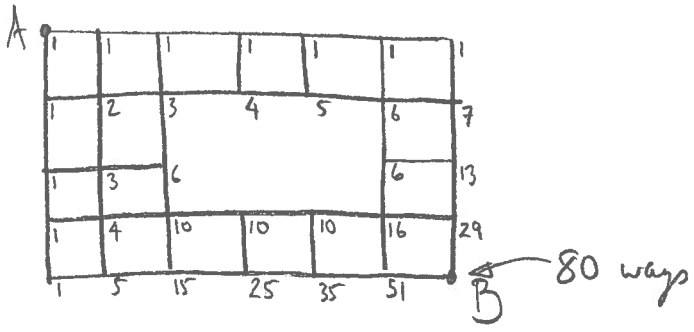
$t_8 = {}_{11}C_7\,(2x)^{11-7}(-y)^7$

$= 330\,(2)^4 x^4 (-y)$

$= -5280\,x^4 y$

⑬

C



80 ways

⑭

C

$n = 12 + 8 + 5 = 25$
$r = 4$

${}_{25}C_4 = 12\,650$

(15) We know all possible ways to pick any 4 vehicles is 12 650

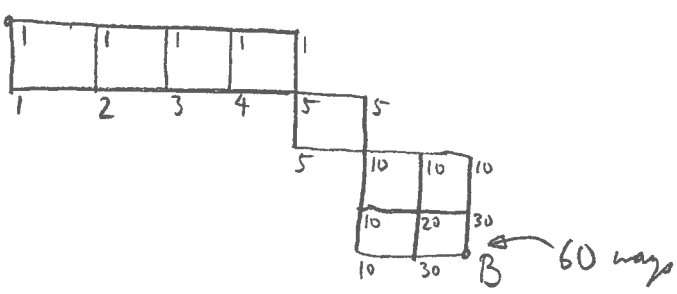
D The # ways to pick NO cars is: $_{13}C_4 = 715$ (since 13 other vehicles other than cars)

Subtract to get # ways to get @ least 1 car: 11 935

(16) $(a+b)^7 \Rightarrow n=7 \Rightarrow 7+1 = 8$ terms

C

(17) A



60 ways

C

(18) $\underset{\text{odd}}{5} \times \underset{\text{odd}}{4} \times \underset{\text{even}}{3} \times \underset{\text{even}}{4} \times \underset{\text{even}}{3} = 720$

digits 1 to 9 inclusive $\Rightarrow$ 9 digits (no repetition)

$\Rightarrow$ 5 odd

$\Rightarrow$ 4 even

(19) $n=14$ (6 girls + 8 boys)

C $r=5$

$_6C_2 \times {}_8C_3 = 840$

(girls) (boys)

(20) 2 cases: 0 boys chosen: $_8C_0 \times {}_6C_5 = 6$

D 1 boy chosen: $_8C_1 \times {}_6C_4 = 120$

AT most 1 boy chosen: 126 ways

(21) $\frac{200!}{198!} = \frac{200(199)(198!)}{198!} = 200 \times 199 = 39800$

C

(22) $\underset{L}{3} \times \underset{D}{10} \times \underset{L}{26} \times \underset{D}{10} \times \underset{L}{26} \times \underset{D}{10} = 2\,028\,000$

D (V, H, nX)

Repeats allowed

26 letters + 10 digits (0-9)

3 mk

(23) 3rd term $\Rightarrow r = 2$ (always 1 less)

C $(x+2y)^7 \Rightarrow n = 7$, $a = x$, $b = 2y$

$t_{r+1} = {}_nC_r\, a^{n-r} b^r$

$\therefore t_3 = {}_7C_2\, x^5 (2y)^2$

$= 21 x^5 (2)^2 y^2$

$= 84 x^5 y^2$

$\therefore$ coefficient is 84

(24) BALLOON

B $n = 7$, $L = 2$, $O = 2$ $\quad \frac{7!}{2!2!} = 1260$

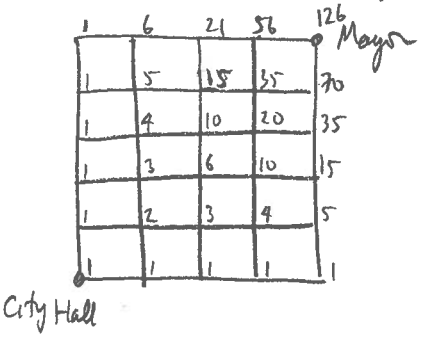
(25) $n = 34$ (20 ♀, 14 ♂)

B $\quad {}_{20}C_5 \cdot {}_{14}C_3 = 15504 \cdot 364 = 5\,643\,456$

(26) PEPPER

A $n = 6$, $P = 3$, $E = 2$ $\quad \frac{6!}{3!2!} = 60$

(27) draw a pick to help



126 ways

(28) $(x+y)^{10}$

D $n = 10$, $a = x$, $b = y$

$t_{r+1} = {}_{10}C_r \; x^{10-r} y^r$ → need $x^{10-r} y^r = x^8 y^2$

$\Rightarrow r = 2$

$\therefore$ coefficient is ${}_{10}C_2 = 45$

(29) D

2 cases: 0 hearts in a 4 card hand is: $39C_4 = 82\,251$

(no hearts means 52 − 13 ♡s in deck = 39)

1 heart in a 4-card hand: $13C_1 \cdot 39C_3 = 118\,807$

(choose 1 of 13 ♡s & 3 of 39 other suits)

So ADD & get at most 1 heart in hand: 201 058

(30) D

$$\underbrace{4 \times 3 \times 2 \times 1}_{\text{Eng}} \times \underbrace{2 \times 1}_{\text{Chem}} \times \underbrace{3 \times 2 \times 1}_{\text{math}} = 288$$

But we can also arrange the order of ENG, Chem, & math in 3! ways.

So do $288 \times 3! = 1728$ ways

Written

① 7 ♂ 5 ♀ so $n = 12$

a) exactly 1 boy: $7C_1 \cdot 5C_3 = 7 \cdot 10 = 70$

& Committee of 4

(Choose 1 boy) (3 girls left to choose)

b) $5C_1 \cdot 7C_1 \cdot 10C_2 = 1575$

(Choose 1 ♀ for pres.) (Choose 1 ♂ for VP) (10 students left)

② a) $7C_3 = 35$

b) $n = 7$, $b = 2$, $y = 3$, $r = 1$, $g = 1$

$$\frac{7!}{2!\,3!\,1!\,1!\,1!} = 420$$

1!'s not necessary

③ a) $n = 13$, $8♂$, $5♀$, $r = 7$

$$\underbrace{{}_8C_3}_{\text{exactly 3 men}} \cdot \underbrace{{}_5C_4}_{\text{choose 4 women to make 7}} = 280$$

b) $n = 13$, $8♂$, $5♀$, $r = 6$

Cases: 4 women in groups: ${}_5C_{\textcircled{4}} \cdot {}_8C_{\textcircled{2}} = 140$ (add to $r = 6$)

5 —"— : ${}_5C_{\textcircled{5}} \cdot {}_8C_{\textcircled{1}} = 8$ (must add to $r = 6$)

148 ← at least 4 women in the group of 6

~~6~~ ← can't do 6 bc only 5 women available!

$\underbrace{\quad}_{♀}$ $\underbrace{\quad}_{♂}$

④ $${}_nP_2 = 90 \implies \frac{n!}{(n-2)!} = 90$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 90$$

$$n(n-1) - 90 = 0$$

$$n^2 - n - 90 = 0$$

$$(n-10)(n+9) = 0$$

$\therefore n = 10$, ~~$n = -9$~~ reject $\because n \in \mathbb{N}$ for $!$ to be defined

Don't believe me? Try $(-9)!$ on your calculator . . .