* Test questions will be very similar to these ( 10 marks M.C. 10 marks written )

**Exponential and Logarithmic Functions Review**

1. Determine the domain of $y = \log(x + 1)$.
2. Determine an equivalent expression for $\log\frac{100a^2}{\sqrt{b}}$.
3. Evaluate: $\log_{\sqrt{7}} 7^3$
4. As an iceberg melts during the summer, it loses 3% of its mass every 5 days. This iceberg reduces to 40% of its original mass after t days. Write an exponential equation to model this situation.
5. Solve: $\log_2(\log_9 x) = -1$
6. Solve: $5^{x+1} = 2(3^{2x})$
7. Change to logarithmic form: $a^3 = b$
8. A population grows continuously according to the formula $P = P_0e^{kt}$, where P is the final population at the end of t years, $P_0$ is the initial population and k is the annual growth rate. What will the population be at the end of 8 years if the initial population is 15 million and the annual growth rate is 4%?
9. Determine the magnitude of an earthquake that is half as intense as an earthquake of magnitude 8.0 on the Richter scale.
10. Solve algebraically: $\log 2 - \log(x + 1) = \log(x + 1) - \log(x + 17)$
11. Change $\log_a p = t$ to exponential form.
12. Determine a simplified expression for: $\log a + 2\log b - 3\log c$
13. Solve algebraically: $\log_5(3x) - \log_5(x - 3) = 2$
14. Solve: $9^{x+2} = (3^{4x-3})(3^5)$
15. If $x = \log_5 3$ and $y = \log_5 4$, express $\log_5 144$ in terms of x and y.
16. What equation represents the graph of $y=2^x$ after it is reflected in the $x$-axis?
17. A ball is dropped from a height of 4 m. After each bounce, the ball rises to 70% of its previous height. What is the maximum height the ball will reach after it hits the ground for the 5th time?
18. A population grows continuously according to the formula $P = P_0e^{kt}$, where P is the final population in t years, $P_0$ is the initial population and k is the continuous growth rate. If the population in 7 years is 27 191 and the initial population is 25 000, find the continuous growth rate.
19. A pH of 5 is 10 times more acidic than a pH of 6. Solution A has a pH of 5.7. Solution B is 1260 times more acidic than Solution A. Find the pH of solution B.
20. A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g? [solve algebraically]
21. If the function $y=3^x$ is expanded vertically by a factor of 9 to produce a new function, write an equation of the new function in the form $y=3^{kx}$. — x+k
22. A particular type of bacteria multiplies 5-fold every 30 minutes. Initially there are 100 bacteria. Determine an expression for the number of bacteria after k minutes.
23. Given $f(x)=2^x+5$, determine $f^{-1}(x)$, the inverse of $f(x)$.
24. Solve algebraically: $2\log_3(x+4) - \log_3(-x) = 2$
25. Jules invests $5000 at an interest rate of 4% per annum, compounded semi-monthly. Write an expression to represent Jules' investment after t years.
26. Determine the half-life of an unknown substance if it takes 120 days for 10% of it to remain.
27. Show that $(\log\frac{a}{b})(\log\frac{c}{d})=(\log\frac{a}{c})(\log\frac{b}{d})+(\log\frac{a}{d})(\log\frac{c}{b})$. State any restrictions.
28. Prove that $\log_b x^n = n\log_b x$ for $b \neq 1$ and $b > 0$.
29. Prove the identity: $\log_b x^n = (n\log_c x) \div (\log_c b)$.
30. For what value(s) of $k$ is $x$ a natural number: $\log_2(x + 1) = \log_4(kx)$.

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① $y = \log(x+1)$  $x+1 > 0 \Rightarrow x > -1$

② $\log \dfrac{100a^2}{\sqrt{b}} = \log 100 + 2\log a + \frac{1}{2}\log b$

③ $\log_{\sqrt{7}} 7^3 = 3\log_{\sqrt{7}} \sqrt{7}^2 = 3 \cdot 2 = 6$

④ $A(t) = A_0 (0.97)^{t/5} \longrightarrow 0.40 = 0.97^{t/5}$

⑤ $\log_2(\log_9 x) = -1 \Rightarrow 2^{\log_2(\log_9 x)} = 2^{-1}$

$\Rightarrow \log_9 x = \frac{1}{2}$

$\Rightarrow x = 9^{\frac{1}{2}} = 3$

⑥ $5^{x+1} = 2(3^{2x}) \Rightarrow \log(5^{x+1}) = \log[2(3^{2x})]$

$\Rightarrow (x+1)\log 5 = \log 2 + 2x\log 3$

$\Rightarrow \log 5 - \log 2 = 2x\log 3 - x\log 5$

$\Rightarrow \log 5 - \log 2 = x(2\log 3 - \log 5)$

$\Rightarrow x = \dfrac{\log 5 - \log 2}{2\log 3 - \log 5} \doteq 1.56$

⑦ $a^3 = b \Rightarrow \log_a b = 3$

⑧ $P = P_0 e^{kt} \Rightarrow P = 15\,000\,000\, e^{0.04(8)}$

$\doteq 20\,656\,916$

⑨ $\dfrac{10^{8.0}}{10^R} = 2 \Rightarrow 10^{8.0-R} = 2 \Rightarrow \log 10^{8.0-R} = \log 2$

$\Rightarrow 8.0 - R = \log 2 \Rightarrow R = 8.0 - \log 2 \doteq 7.7$

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(10) $\log 2 - \log(x+1) = \log(x+1) - \log(x+17)$

$\Rightarrow \log 2 + \log(x+17) = \log(x+1) + \log(x+1)$

$\Rightarrow \log\left[2(x+17)\right] = \log\left[(x+1)(x+1)\right]$

$\Rightarrow 2(x+17) = (x+1)(x+1)$

$2x + 34 = x^2 + 2x + 1$

$x^2 = 33$

$x = \sqrt{33}$ $(-\sqrt{33}$ inadmissable$)$

(11) $\log_a p = t \Rightarrow p = a^t$

(12) $\log a + 2\log b - 3\log c = \log \frac{ab^2}{c^3}$

(13) $\log_5(3x) - \log_5(x-3) = 2$

$\Rightarrow \log_5\left(\frac{3x}{x-3}\right) = \log_5(5^2)$

$\Rightarrow \frac{3x}{x-3} = 25$

$\Rightarrow 3x = 25x - 75$

$\Rightarrow 22x = 75$

$\Rightarrow x = \frac{75}{22}$

(14) $9^{x+2} = 3^{4x-3} \cdot 3^5$

$\Rightarrow 3^{3(x+2)} = 3^{(4x-3)+5}$

$\Rightarrow 3(x+2) = (4x-3)+5$ (same bases)

$\Rightarrow 3x + 6 = 4x + 2$

$\Rightarrow x = 4$

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(15) $x = \log_5 3$ & $y = \log_5 4 \Rightarrow \log_5 144 = \log_5 12^2$

$= 2\log_5(3 \times 4)$

$= 2(\log_5 3 + \log_5 4)$

$= 2(x+y)$

(16) $y = 2^x$ reflected in x-axis is $y = -2^x$ (graph to check!)

(17) $h(b) = h_0(0.70)^b$, $h_0$ = initial height (m), $b$ = # bounces, $h(t)$ = max height @ b

$h(5) = 4(0.70)^5 \doteq 0.67$ m

(18) $P = P_0 e^{kt}$ $\quad 27191 = 25000\, e^{r(7)}$

$\therefore 1.8764 = e^{7r}$

$\ln 1.8764 = 7r$

$\therefore r = \frac{\ln 1.8764}{7} \doteq 0.0899$ or $9.0\%$

(19) Solution A has pH 5.7
Solution B has pH a

$\frac{10^{5.7}}{10^a} = 1260 \quad \therefore 10^{5.7-a} = 1260$

$\log 10^{5.7-a} = \log 1260$

$5.7 - a = \log 1260$

* recall a lower pH value is more acidic

$\therefore a = 5.7 - \log 1260 \doteq 2.6$ is pH of B

(20) $\frac{95}{300} = \frac{\cancel{300}(\frac{1}{2})^{t/17}}{\cancel{300}} \Rightarrow \frac{19}{60} = \left(\frac{1}{2}\right)^{t/17}$

$\log \frac{19}{60} = \frac{t}{17} \log \frac{1}{2}$

$\therefore t = \frac{17 \log \frac{19}{60}}{\log \frac{1}{2}} \doteq 28.2$ d

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$*$ error in question — of form $y = 3^{x+k}$ (not $y = 3^{kx}$)

(21) $y = 3^x$ expanded vertically by factor of 9 $\Rightarrow y = 9 \cdot 3^x = 3^2 \cdot 3^x = 3^{x+2}$

$\Rightarrow y = 3^{x+2}$ $*$

(22) $B(k) = 100(5)^{k/30}$, $B(k)$ = # bacteria after $k$ minutes.

(23) $f(x) = 2^x + 5$ let $x = 2^y + 5 \Rightarrow x - 5 = 2^y$

$\Rightarrow \log_2(x-5) = \log_2 2^y$

$\Rightarrow y = \log_2(x-5)$

$\therefore f^{-1}(x) = \log_2(x-5)$

(24) $2\log_3(x+4) - \log_3(-x) = 2$

$x + 4 > 0 \Rightarrow x > -4$, $-x > 0 \Rightarrow x < 0$ $\Big\}$ $-4 < x < 0$ domain of $x$

$\Rightarrow \log_3\left[\frac{(x+4)^2}{-x}\right] = \log_3(3^2)$

$\Rightarrow \frac{(x+4)^2}{-x} = 9$

$\Rightarrow x^2 + 8x + 16 = -9x$

$\Rightarrow x^2 + 17x + 16 = 0$

$\Rightarrow (x+1)(x+16) = 0 \Rightarrow x = -1$ & $\cancel{x = -16}$ inadmissable (not in domain) $\therefore x = -1$

(25) $I(t) = 5000\left(1 + \frac{0.04}{24}\right)^{24t}$

(26) $0.10\cancel{A_0} = \cancel{A_0}\left(\frac{1}{2}\right)^{\frac{120}{T}}$ $A_0$ is initial amount & $T$ is the half-life in days

$\log 0.10 = \frac{120}{T}\log\frac{1}{2}$

$T \cdot \log 0.10 = 120 \log\frac{1}{2}$

$\therefore T = \frac{120\log\frac{1}{2}}{\log 0.10}$

$\doteq 36.1$ days

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(27) Show $\left(\log \frac{a}{b}\right)\left(\log \frac{c}{d}\right) = \left(\log \frac{a}{c}\right)\left(\log \frac{b}{d}\right) + \left(\log \frac{a}{d}\right)\left(\log \frac{c}{b}\right)$

$LS = \left(\log \frac{a}{b}\right)\left(\log \frac{c}{d}\right)$

$= (\log a - \log b)(\log c - \log d)$

$= \log a \log c - \log a \log d - \log b \log c + \log b \log d$

$RS = \left(\log \frac{a}{c}\right)\left(\log \frac{b}{d}\right) + \left(\log \frac{a}{d}\right)\left(\log \frac{c}{b}\right)$

$= (\log a - \log c)(\log b - \log d) + (\log a - \log d)(\log c - \log b)$

$= \cancel{\log a \log b} - \log a \log d - \log b \log c + \cancel{\log c \log d}$

$+ \log a \log c - \cancel{\log a \log b} - \cancel{\log c \log d} + \log b \log d$

$= \log a \log c - \log a \log d - \log b \log c + \log b \log d$

$\because LS = RS$

$\therefore \left(\log \frac{a}{b}\right)\left(\log \frac{c}{d}\right) = \left(\log \frac{a}{c}\right)\left(\log \frac{b}{d}\right) + \left(\log \frac{a}{d}\right)\left(\log \frac{c}{b}\right)$ where $a, b, c, d > 0$ above

(28) Prove $\log_b x^n = n \log_b x$ let $y = \log_b x \Rightarrow x = b^y$

then $\log_b x^n = \log_b (b^y)^n$

$= \log_b b^{(ny)}$

$= ny$

$= n \log_b x$

(29) Prove $\log_b x^n = \frac{n \log_c x}{\log_c b}$ let $y = \log_b x^n \Rightarrow x^n = b^y$

then $\log_c x^n = \log_c b^y$

$n \log_c x = y \log_c b$ (using #28)

$y = \frac{n \log_c x}{\log_c b}$

$\therefore \log_b x^n = \frac{n \log_c x}{\log_c b}$

CHALLENGE

(30) $\log_2 (x+1) = \log_4 (kx) = \frac{\log_2 (kx)}{\log_2 4} = \frac{\log_2 kx}{\log_2 2^2} = \frac{1}{2} \log_2 kx = \log_2 \sqrt{kx}$

$\Rightarrow x + 1 = \sqrt{kx} \Rightarrow (x+1)^2 = kx \Rightarrow x^2 + 2x + 1 = kx \Rightarrow x^2 + (2-k)x + 1 = 0$

$\Rightarrow 2 - k = -2$ or $2 - k = 2 \Rightarrow k = 4$ or $\cancel{k = 0}$ $\therefore k = 4$