Exponential and Logarithmic Functions Review

1. Determine the domain of $y = \log(x + 1)$.
2. Determine an equivalent expression for $\log\frac{100a^2}{\sqrt{b}}$.
3. Evaluate: $\log_{\sqrt{7}} 7^3$
4. As an iceberg melts during the summer, it loses 3% of its mass every 5 days. This iceberg reduces to 40% of its original mass after t days. Write an exponential equation to model this situation.
5. Solve: $\log_2(\log_9 x) = -1$
6. Solve: $5^{x+1} = 2(3^{2x})$
7. Change to logarithmic form: $a^3 = b$
8. A population grows continuously according to the formula $P = P_0e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population and k is the annual growth rate. What will the population be at the end of 8 years if the initial population is 15 million and the annual growth rate is 4%?
9. Determine the magnitude of an earthquake that is half as intense as an earthquake of magnitude 8.0 on the Richter scale.
10. Solve algebraically: $\log 2 - \log(x + 1) = \log(x + 1) - \log(x + 17)$
11. Change $\log_a p = t$ to exponential form.
12. Determine a simplified expression for: $\log a + 2\log b - 3\log c$
13. Solve algebraically: $\log_5(3x) - \log_5(x - 3) = 2$
14. Solve: $9^{x+2} = (3^{4x-3})(3^5)$
15. If $x = \log_5 3$ and $y = \log_5 4$, express $\log_5 144$ in terms of x and y.
16. What equation represents the graph of $y=2^x$ after it is reflected in the x-axis?
17. A ball is dropped from a height of 4 m. After each bounce, the ball rises to 70% of its previous height. What is the maximum height the ball will reach after it hits the ground for the 5th time?
18. A population grows continuously according to the formula $P = P_0e^{kt}$, where P is the final population in t years, P_0 is the initial population and k is the continuous growth rate. If the population in 7 years is 27 191 and the initial population is 25 000, find the continuous growth rate.
19. A pH of 5 is 10 times more acidic than a pH of 6. Solution A has a pH of 5.7. Solution B is 1260 times more acidic than Solution A. Find the pH of solution B.
20. A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g? [solve algebraically]
21. If the function $y=3^x$ is expanded vertically by a factor of 9 to produce a new function, write an equation of the new function in the form $y=3^{x+k}$.
22. A particular type of bacteria multiplies 5-fold every 30 minutes. Initially there are 100 bacteria. Determine an expression for the number of bacteria after k minutes.
23. Given $f(x)=2^x+5$, determine $f^{-1}(x)$, the inverse of $f(x)$.
24. Solve algebraically: $2\log_3(x+4) - \log_3(-x) = 2$
25. Jules invests $5000 at an interest rate of 4% per annum, compounded semi-monthly. Write an expression to represent Jules' investment after t years.
26. Determine the half-life of an unknown substance if it takes 120 days for 10% of it to remain.
27. Show that $(\log\frac{a}{b})(\log\frac{c}{d})=(\log\frac{a}{c})(\log\frac{b}{d})+(\log\frac{a}{d})(\log\frac{c}{b})$. State any restrictions.
28. Prove that $\log_b x^n = n\log_b x$ for $b \neq 1$ and $b > 0$.
29. Prove the identity: $\log_b x^n = (n\log_c x) \div (\log_c b)$.
30. For what value(s) of k is x a natural number: $\log_2(x + 1) = \log_4(kx)$.