Principles of Mathematics 12

Sample 2007/08

Provincial Examination — Multiple-Choice Key

Cognitive Processes	Weightings	Question Type
K = Knowledge	10%	**44** = Multiple Choice (MC)
U = Understanding	70%	**8** = Written Response (WR)
H = Higher Mental Processes	20%	

Topics	Prescribed Learning Outcomes (PLOs)	
1. Patterns and Relations	A	A1–A3: Geometric Sequences and Series
2. Shape and Space	B	A4–A6, A13–15: Logarithms and Exponents
3. Statistics and Probability	C	A7–A12, A16-A18: Trigonometry
		B1–B6: Shape and Space
		C1–C5: Combinatorics
		C5–C10: Probability

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	C	U	1.5	1	A8	MC
2.	D	U	1.5	1	A17	MC
3.	D	K	1.5	1	A16	MC
4.	C	U	1.5	1	A17	MC
5.	A	U	1.5	1	A17	MC
6.	D	U	1.5	1	A12	MC
7.	B	U	1.5	1	A9	MC
8.	C	H	1.5	1	A9, A12	MC
9.	D	K	1.5	1	A15	MC
10.	B	U	1.5	1	A6	MC
11.	C	U	1.5	1	A6	MC
12.	A	U	1.5	1	A14	MC
13.	C	H	1.5	1	A5	MC
14.	D	H	1.5	1	A5	MC
15.	A	U	1.5	3	C5	MC
16.	C	K	1.5	2	B1, B2	MC
17.	A	U	1.5	2	B3	MC
18.	A	U	1.5	2	B1, B3	MC
19.	B	U	1.5	2	B1, B2	MC
20.	A	H	1.5	2	B1, B3	MC
21.	C	U	1.5	2	B1, B2	MC
22.	B	U	1.5	1	A9	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
23.	A	U	1.5	1	A18	MC
24.	C	U	1.5	1	A7	MC
25.	C	H	1.5	1	A11	MC
26.	D	K	1.5	1	A1	MC
27.	B	U	1.5	1	A1	MC
28.	C	U	1.5	1	A2	MC
29.	B	U	1.5	1	A3	MC
30.	C	U	1.5	1	A1	MC
31.	D	H	1.5	1	A1	MC
32.	A	K	1.5	1	A13	MC
33.	A	U	1.5	1	A14	MC
34.	C	U	1.5	1	A15	MC
35.	C	U	1.5	3	C1	MC
36.	A	U	1.5	3	C3	MC
37.	D	U	1.5	3	C4	MC
38.	D	U	1.5	3	C5	MC
39.	B	U	1.5	3	C1, C3	MC
40.	A	U	1.5	3	C6	MC
41.	C	U	1.5	3	C8	MC
42.	A	H	1.5	3	C8	MC
43.	B	U	1.5	3	C10	MC
44.	C	U	1.5	3	C5	MC

Principles of Mathematics 12

Sample 2007/08

Provincial Examination — Written-Response Key

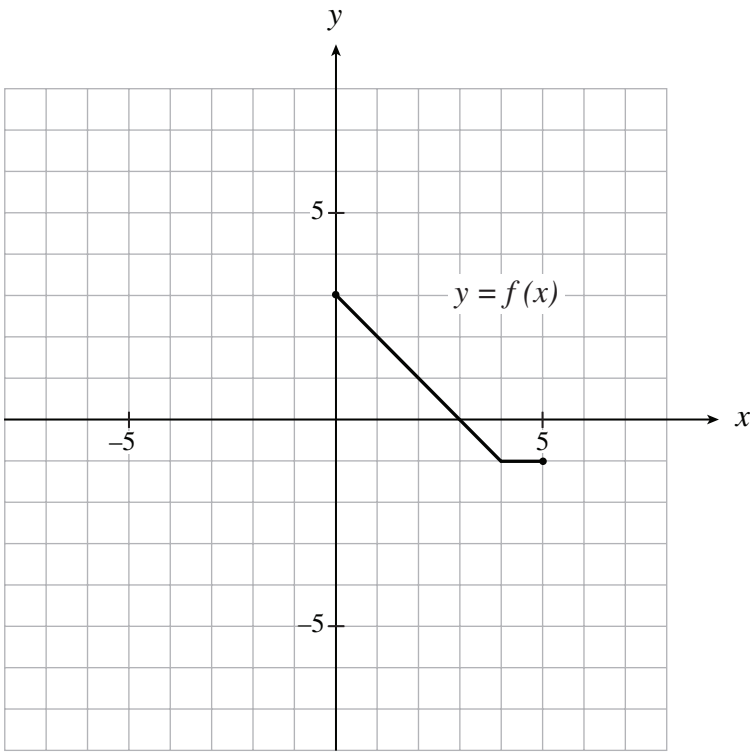
Cognitive Processes	Weightings	Question Type
K = Knowledge	10%	**44** = Multiple Choice (MC)
U = Understanding	70%	**8** = Written Response (WR)
H = Higher Mental Processes	20%	

Topics	Prescribed Learning Outcomes (PLOs)	
1. Patterns and Relations	A	A1–A3: Geometric Sequences and Series
2. Shape and Space	B	A4–A6, A13–15: Logarithms and Exponents
3. Statistics and Probability	C	A7–A12, A16-A18: Trigonometry
		B1–B6: Shape and Space
		C1–C5: Combinatorics
		C5–C10: Probability

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	–	U	3	2	B5, B6	WR
2.	–	U	2	2	B4	WR
3.	–	U	5	1	A5	WR
4.	–	U	3	3	C10	WR
5.	–	U	2	3	C10	WR
6.	–	U	3	1	A18	WR
7.	–	U	1	1	A10	WR
8.	–	H	5	1	A11, A12	WR

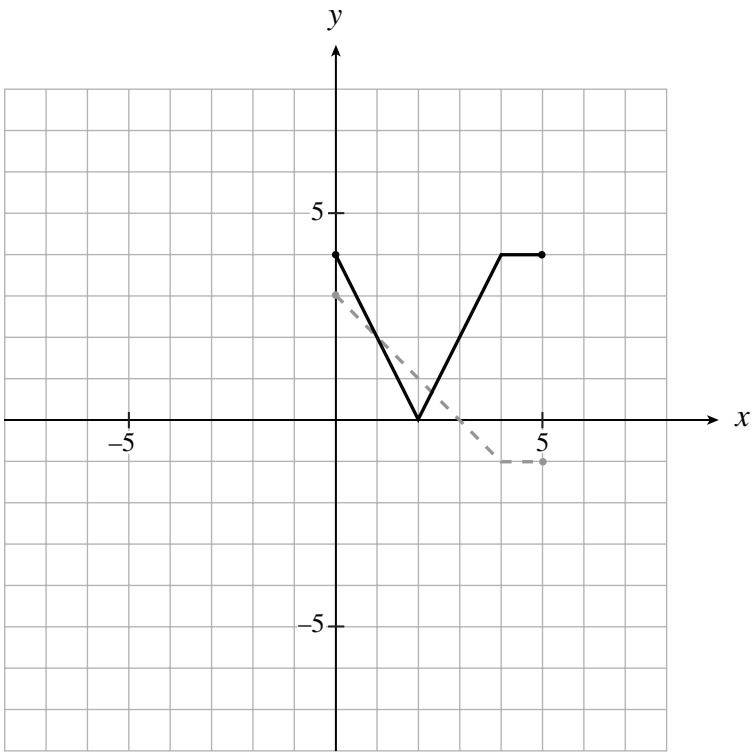
Use the following graph to answer questions 1 and 2.

The graph of $y = f(x)$ is shown below.



1. On the grid provided, sketch the graph of $y = 2\left| f(x) - 1 \right|$. **(3 marks)**

SOLUTION _____



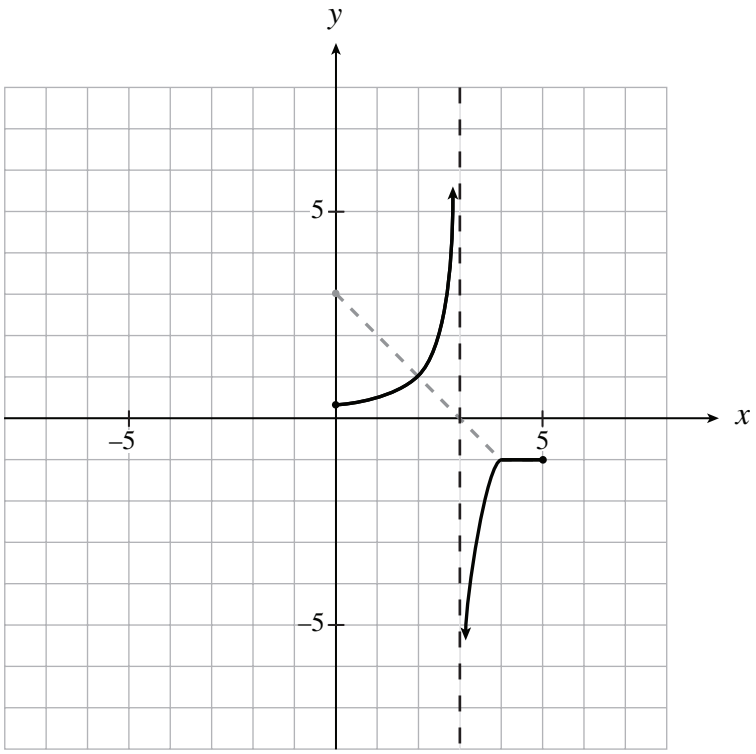
1 mark: vertical translation
1 mark: absolute value
1 mark: vertical expansion

Note: **Deduct $\frac{1}{2}$ mark if graph does not end at correct points**

Cap at 2 marks if wrong order of transformations.

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

SOLUTION



$\frac{1}{2}$ mark: asymptotic behaviour

1 mark: invariant points

$\frac{1}{2}$ mark: shape from $x = 0$ to $x = 2$

Note: **Deduct $\frac{1}{2}$ mark if graph does not end at correct points $\left(0, \frac{1}{3}\right)$ and $(5, -1)$**

3. Solve algebraically $\log 2 - \log(x-1) = \log(x+1) - \log(x+17)$. **(5 marks)**

SOLUTION

$$\log \frac{2}{x-1} = \log \frac{x+1}{x+17} \quad \leftarrow \textbf{1 mark}$$

$$\frac{2}{x-1} = \frac{x+1}{x+17} \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$2x + 34 = x^2 - 1 \quad \leftarrow \textbf{1 mark}$$

$$0 = x^2 - 2x - 35 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$0 = (x-7)(x+5)$$

$$x = 7, \quad x = -5 \quad \leftarrow \tfrac{1}{2} \textbf{ mark}$$

$$\uparrow \qquad \downarrow$$

1 mark reject $\leftarrow \tfrac{1}{2}$ **mark**

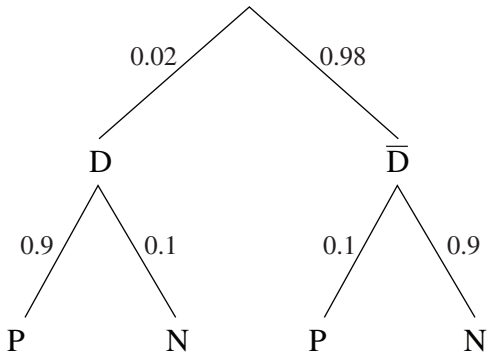
$$\therefore \boxed{x = 7}$$

Use the following information to answer questions 4 and 5.

It is known that 2% of the population has a certain disease. A test for this disease is 90% accurate. This means that the outcome of the test is correct 90% of the time. A positive test result claims that a person has the disease.

4. Determine the probability that a randomly selected person will test positive for this disease.
   (Answer accurate to at least 3 decimal places.) **(3 marks)**

SOLUTION



← **1 mark tree diagram**

$\frac{1}{2}$ mark ↓ **$\frac{1}{2}$ mark** ↓

$$P(P) = (0.9)(0.02) + (0.1)(0.98)$$

$$P(P) = 0.116$$

or

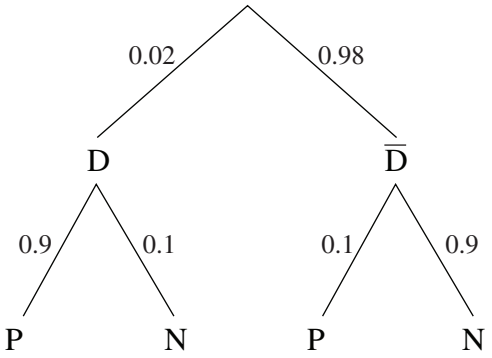
$$\frac{29}{250}$$

← **1 mark**

5. Given that a randomly selected person tests positive, what is the probability that this person actually has the disease?

   (Answer accurate to at least 2 decimal places.) **(2 marks)**

SOLUTION _____



$$P(D|P) = \frac{P(D \text{ and } P)}{P(P)}$$

$$P(D|P) = \frac{(0.02)(0.9)}{0.1160} \begin{matrix} \leftarrow \frac{1}{2} \textbf{ mark} \\ \leftarrow \frac{1}{2} \textbf{ mark} \end{matrix} \quad \frac{1}{2} \textbf{ mark}$$

$$P(D|P) = 0.16$$

or

$$\frac{9}{58}$$

$\leftarrow \frac{1}{2}$ **mark**

6. A minimum value of a sinusoidal function is at $\left(\frac{\pi}{4},\ 3\right)$. The nearest maximum value to the right of this point is at $\left(\frac{7\pi}{12},\ 7\right)$. Determine an equation of this function. **(3 marks)**

SOLUTION

$\frac{1}{2}$ **mark** $\frac{1}{2}$ **mark**

$$y = 2\sin 3\left(x - \frac{5\pi}{12}\right) + 5$$

1 mark **1 mark**

OR

$$y = -2\cos 3\left(x - \frac{\pi}{4}\right) + 5$$

7. The two smallest positive solutions of $\cos 4x = 0.6$ are $x = 0.23$ and $x = 1.34$.
Determine the general solution for $\cos 4x = 0.6$. **(1 mark)**

SOLUTION _____

$$x = 0.23 + \frac{\pi n}{2}, \quad 1.34 + \frac{\pi n}{2}, \quad \text{where } n \text{ is an integer}$$

$\uparrow$ $\frac{1}{2}$ **mark** $\uparrow$ $\frac{1}{2}$ **mark**

Note: **Deduct $\frac{1}{2}$ mark for not writing "where *n* is an integer"**

8. Prove the identity: **(5 marks)**

$$\frac{\tan x}{\sec x + 1} = \frac{2\cos x - 2\cos^2 x}{\sin 2x}$$

SOLUTION

LEFT SIDE	RIGHT SIDE
$\dfrac{\tan x}{\sec x + 1}$	$\dfrac{2\cos x(1-\cos x)}{2\sin x\cos x} \leftarrow \frac{1}{2}$ **mark** $\leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\cos x}{\cos x}\dfrac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}+1\right)}$ ($\frac{1}{2}$ **mark** $\downarrow$)	$\dfrac{1-\cos x}{\sin x} \leftarrow \frac{1}{2}$ **mark**
$\frac{1}{2}$ **mark** $\rightarrow$ $\dfrac{\sin x}{1+\cos x}$	$\left(\dfrac{1-\cos x}{\sin x}\right)\left(\dfrac{1+\cos x}{1+\cos x}\right) \leftarrow \frac{1}{2}$ **mark**
	$\dfrac{1-\cos^2 x}{\sin x(1+\cos x)} \leftarrow \frac{1}{2}$ **mark**
	$\dfrac{\sin^2 x}{\sin x(1+\cos x)} \leftarrow \frac{1}{2}$ **mark**
	$\dfrac{\sin x}{1+\cos x} \leftarrow \frac{1}{2}$ **mark**

LS = RS