Principles of Mathematics 12

August 2006 — Form A

Provincial Examination — Answer Key

Cognitive Processes	Weightings	Question Type
K = Knowledge	10%	**44** = Multiple Choice (MC)
U = Understanding	70%	**6** = Written Response (WR)
H = Higher Mental Processes	20%	

Topics	Prescribed Learning Outcomes (PLOs)
1. Problem Solving	A
2. Patterns and Relations	B, C, D
3. Shape and Space	E, F
4. Statistics and Probability	G

A	Problem Solving and Cross Topic Problems
B	Geometric Sequences and Series
C/D	Logarithms and Exponents
C/D	Trigonometry
E	Conics
F	Transformations
G	Combinatorics
G	Probability
G	Statistics

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	D	K	1.5	2	D6	MC
2.	B	U	1.5	2	C3	MC
3.	D	U	1.5	2	D6	MC
4.	A	U	1.5	2	D6	MC
5.	B	U	1.5	2	C8	MC
6.	A	H	1.5	2	C5	MC
7.	B	K	1.5	2	D2	MC
8.	A	U	1.5	2	D4	MC
9.	C	U	1.5	2	C2	MC
10.	B	H	1.5	2	C1	MC
11.	A	U	1.5	3	E2	MC
12.	A	U	1.5	3	E2	MC
13.	D	H	1.5	3	E2	MC
14.	B	H	1.5	3	E3	MC
15.	A	U	1.5	3	F1	MC
16.	B	U	1.5	3	F3	MC
17.	B	K	1.5	3	F2	MC
18.	A	U	1.5	3	F2	MC
19.	B	H	1.5	3	F6	MC
20.	C	K	1.5	3	E1	MC
21.	D	U	1.5	3, 1	E2; F3; A2	MC
22.	C	U	1.5	2	D5	MC
23.	C	H	1.5	1, 2	A9, D7	MC

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
24.	A	U	1.5	2	B3	MC
25.	A	U	1.5	2	B1	MC
26.	C	U	1.5	2	B1	MC
27.	B	U	1.5	2	B1	MC
28.	D	H	1.5	2	B1	MC
29.	A	U	1.5	2	D3	MC
30.	A	H	1.5	2	D3	MC
31.	A	U	1.5	4	G6	MC
32.	B	U	1.5	4	G4	MC
33.	D	U	1.5	4	G8	MC
34.	D	U	1.5	4	G7	MC
35.	D	H	1.5	4	G5, G6	MC
36.	B	U	1.5	4	G10	MC
37.	C	U	1.5	4	G13	MC
38.	D	H	1.5	4	G8	MC
39.	C	U	1.5	4	G2	MC
40.	C	U	1.5	4	G1	MC
41.	B	H	1.5	4	G2	MC
42.	D	U	1.5	4	G2	MC
43.	C	U	1.5	4	G2	MC
44.	B	U	1.5	4	G3	MC

Principles of Mathematics 12

August 2006

Provincial Examination — Written-Response Key / Scoring Guide

Cognitive Processes	Weightings	Question Types
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U = Understanding	70%	**6** = Written Response (WR)
H = Higher Mental Processes	20%	

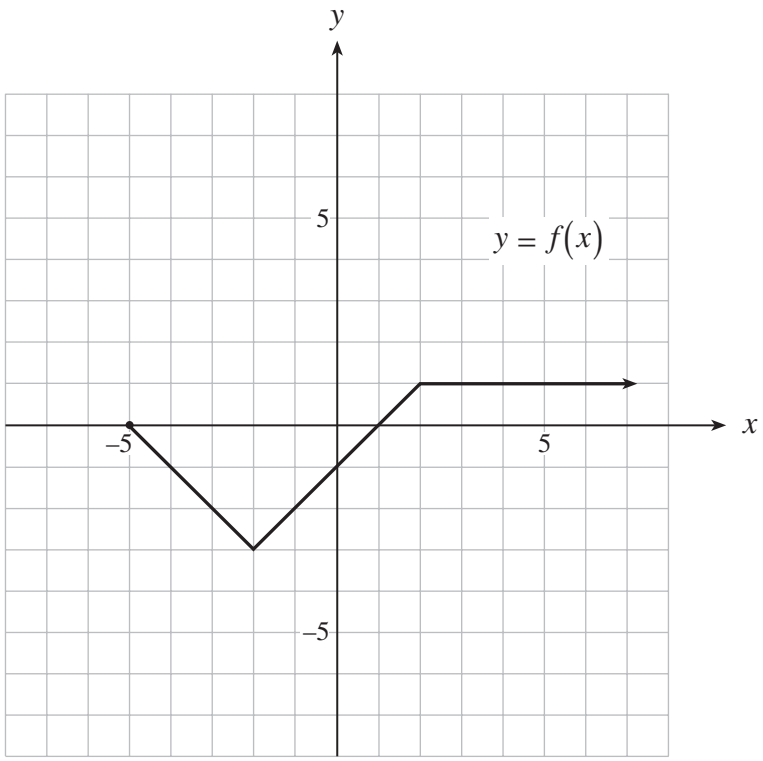
Topics	Prescribed Learning Outcomes (PLOs)
1. Problem Solving	A
2. Patterns and Relations	B, C, D
3. Shape and Space	E, F
4. Statistics and Probability	G

A	Problem Solving and Cross Topic Problems
B	Geometric Sequences and Series
C/D	Logarithms and Exponents
C/D	Trigonometry
E	Conics
F	Transformations
G	Combinatorics
G	Probability
G	Statistics

Question Number	Keyed Response	Cognitive Process	Mark	Topic	PLO	Question Type
1.	–	U	3	3	F5, F6	WR
2.	–	U	2	3	F4	WR
3.	–	U	5	2	D1	WR
4.	–	U	5	4	G12	WR
5.	–	U	4	2	C5, C6	WR
6.	–	H	5	2	C7, C8	WR

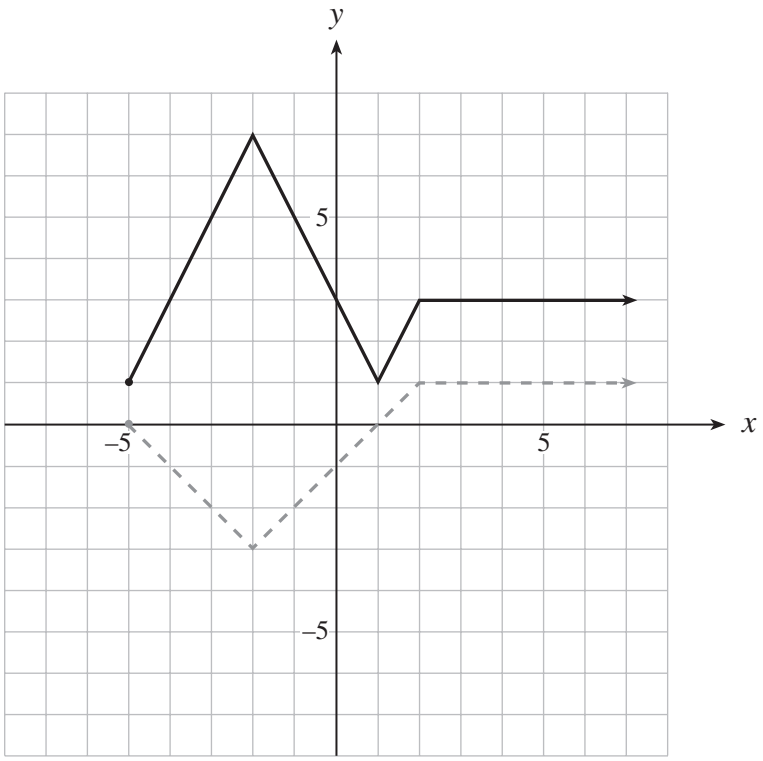
Use the following graph to answer questions 1 and 2.

The graph of $y = f(x)$ is shown below.



1. On the grid provided, sketch the graph of $y = 2\left|\, f(x) \,\right| + 1$. **(3 marks)**

solution

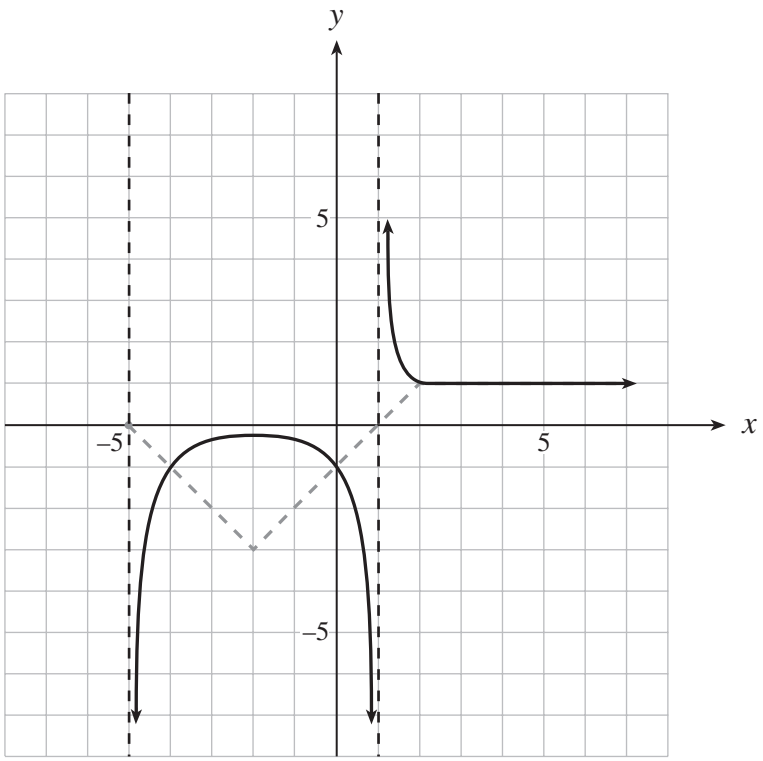


1 mark: absolute value
1 mark: vertical expansion
1 mark: vertical translation

Note: deduct $\frac{1}{2}$ mark if graph does not stop on the left side or does not continue on the right side

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

solution



$\frac{1}{2}$ mark: invariant points – each half

$\frac{1}{2}$ mark: shape – each half

3. A radioactive substance has a half-life of 17 d. How long will it take for 300 g of this substance to decay to 95 g?

   (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.) **(5 marks)**

solution

$$95 = 300\left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \textbf{2 marks}$$

$$\frac{95}{300} = \left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\log\left(\frac{95}{300}\right) = \log\left(\frac{1}{2}\right)^{\frac{d}{17}} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\log\left(\frac{95}{300}\right) = \frac{d}{17}\log\left(\frac{1}{2}\right) \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

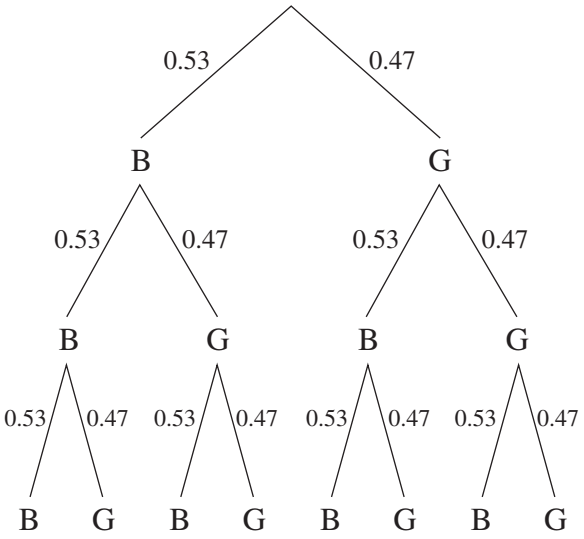
$$d = \frac{17\log\left(\frac{95}{300}\right)}{\log\left(\frac{1}{2}\right)} \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$d = 28.20 \text{ days} \qquad \leftarrow \textbf{1 mark}$$

4. It is known that 53% of graduating students are boys. Three grads are chosen at random. Given that at least two of the three grads are boys, determine the probability that all three of the grads are boys.

   (Answer accurate to at least 4 decimal places.) **(5 marks)**

solution



OR

$$P(\text{3 boys} \mid \text{at least 2 boys})$$

$$= \frac{P(\text{3 boys and at least 2 boys})}{P(\text{at least 2 boys})}$$

$$= \frac{0.53^3 \quad \leftarrow \textbf{1 mark}}{3(0.53)^2(0.47) + (0.53)^3} \xleftarrow{\quad} \textbf{1 mark} \quad \leftarrow \textbf{2 marks}$$

$$= 0.2732 \quad \leftarrow \textbf{1 mark}$$

$$P(\text{3 boys} \mid \text{at least 2 boys})$$

$$= \frac{P(\text{3 boys and at least 2 boys})}{P(\text{at least 2 boys})}$$

$$= \frac{\text{binompdf}\,(3,\ 0.53,\ 3) \quad \leftarrow \textbf{1 mark}}{1 - \text{binomcdf}\,(3,\ 0.53,\ 1)} \xleftarrow{\quad} \textbf{1 mark} \quad \leftarrow \textbf{2 marks}$$

$$= 0.2732 \quad \leftarrow \textbf{1 mark}$$

5. Determine the general solution algebraically. (Solve over the set of real numbers.)

$$3\cos^2 x - 8\cos x + 4 = 0$$

(Answer accurate to at least 2 decimal places.) **(4 marks)**

solution

$$(3\cos x - 2)(\cos x - 2) = 0$$

$\frac{1}{2}$ **mark** $\rightarrow$ $\cos x = \frac{2}{3}$ $\qquad$ $\cos x = 2$ $\leftarrow$ $\frac{1}{2}$ **mark**

$\Downarrow$

$x = 0.84,\ x = 5.44$ $\qquad$ reject $\leftarrow$ $\frac{1}{2}$ **mark**

$\uparrow$ $\qquad$ $\uparrow$

$\frac{1}{2}$ **mk** $\qquad$ **1 mk**

$x = 0.84 + 2n\pi$, $\quad x = 5.44 + 2n\pi$, $\quad n$ is an integer

$\uparrow$ $\qquad$ $\uparrow$

$\frac{1}{2}$ **mk** $\qquad$ $\frac{1}{2}$ **mk**

6. Prove the identity: **(5 marks)**

$$\frac{\tan x\left(\cos x+\cot x\right)}{\sec x+\tan x} = \frac{\sin x \sin 2x}{2-2\cos^2 x}$$

solution

LEFT SIDE		RIGHT SIDE	
	$\dfrac{\tan x\left(\cos x+\cot x\right)}{\sec x+\tan x}$	$\dfrac{\sin x \sin 2x}{2-2\cos^2 x}$	
½ mark →	$\dfrac{\dfrac{\sin x}{\cos x}\left(\cos x+\dfrac{\cos x}{\sin x}\right)}{\dfrac{1}{\cos x}+\dfrac{\sin x}{\cos x}}$	$\dfrac{\sin x\left(2\sin x\cos x\right)}{2\left(1-\cos^2 x\right)}$	← **½ mark**
½ mark →			← **½ mark**
½ mark →	$\dfrac{\sin x+1}{\dfrac{1+\sin x}{\cos x}}$	$\dfrac{2\sin^2 x\cos x}{2\sin^2 x}$	← **½ mark**
1 mark →	$\left(\sin x+1\right)\left(\dfrac{\cos x}{1+\sin x}\right)$	$\cos x$	← **½ mark**
½ mark →	$\cos x$		

LS = RS

6. Prove the identity: **(5 marks)**

$$\frac{\tan x(\cos x + \cot x)}{\sec x + \tan x} = \frac{\sin x \sin 2x}{2 - 2\cos^2 x}$$

alternate solution

LEFT SIDE	RIGHT SIDE
$\dfrac{\tan x(\cos x + \cot x)}{\sec x + \tan x}$	$\dfrac{\sin x \sin 2x}{2 - 2\cos^2 x}$
½ mark → $\dfrac{\dfrac{\sin x}{\cos x}\left(\cos x + \dfrac{\cos x}{\sin x}\right)}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$	$\dfrac{\sin x(2\sin x \cos x)}{2(1 - \cos^2 x)}$ ← **½ mark** ← **½ mark**
½ mark →	$\dfrac{2\sin^2 x \cos x}{2\sin^2 x}$ ← **½ mark**
↓ **½ mark** $\dfrac{\cos x}{\cos x}\dfrac{(\sin x + 1)}{\left(\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}\right)}$	$\cos x$ ← **½ mark**
1 mark → clearing the complex fraction → $\dfrac{\cos x(\sin x + 1)}{1 + \sin x}$	
½ mark → $\cos x$	

LS = RS

END OF KEY