# Principles of Mathematics 12

August 2005 Provincial Examination

## ANSWER KEY / SCORING GUIDE

**CURRICULUM:**

| Organizers | | Sub-Organizers |
|---|---|---|
| 1. Problem Solving | A | Problem Solving and Cross Topic Problems |
| 2. Patterns and Relations | B | Geometric Sequences and Series |
| | C/D | Logarithms and Exponents |
| | C/D | Trigonometry |
| 3. Shape and Space | E | Conics |
| | F | Transformations |
| 4. Statistics and Probability | G | Combinatorics |
| | G | Probability |
| | G | Statistics |

**Part A: Multiple Choice**

| Q | K | C | S | CO | PLO | Q | K | C | S | CO | PLO |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | C | K | 1.5 | 2 | D6 | 23. | C | H | 1.5 | 2 | B1; G7; A2 |
| 2. | A | U | 1.5 | 2 | C4 | 24. | A | K | 1.5 | 2 | D2 |
| 3. | A | U | 1.5 | 2 | C8 | 25. | D | U | 1.5 | 2 | D1 |
| 4. | A | U | 1.5 | 2 | D6; A1 | 26. | D | U | 1.5 | 2 | D2; F3; A2 |
| 5. | D | U | 1.5 | 2 | D5 | 27. | D | K | 1.5 | 3 | F1 |
| 6. | D | H | 1.5 | 2 | C6, C7 | 28. | A | U | 1.5 | 3 | F5 |
| 7. | A | U | 1.5 | 2 | D4 | 29. | B | U | 1.5 | 3 | F3; A1 |
| 8. | B | K | 1.5 | 2 | D3; A2 | 30. | D | H | 1.5 | 3 | F6 |
| 9. | A | H | 1.5 | 2 | D4; A1 | 31. | D | U | 1.5 | 3 | F6 |
| 10. | C | H | 1.5 | 2 | D1; A2; F2 | 32. | C | U | 1.5 | 4 | G5 |
| 11. | C | K | 1.5 | 3 | E2 | 33. | D | U | 1.5 | 4 | G5 |
| 12. | D | K | 1.5 | 3 | E2 | 34. | C | U | 1.5 | 4 | G8 |
| 13. | C | U | 1.5 | 3 | E2; A1 | 35. | B | U | 1.5 | 4 | G6 |
| 14. | D | U | 1.5 | 3 | E2 | 36. | B | U | 1.5 | 4 | G7 |
| 15. | B | U | 1.5 | 3 | E3 | 37. | B | U | 1.5 | 4 | G13 |
| 16. | A | H | 1.5 | 3 | E2 | 38. | A | U | 1.5 | 4 | G11 |
| 17. | B | U | 1.5 | 2 | C3 | 39. | D | H | 1.5 | 4 | G12 |
| 18. | B | U | 1.5 | 2 | C6 | 40. | A | U | 1.5 | 4 | G1; A1 |
| 19. | C | U | 1.5 | 2 | B1 | 41. | C | U | 1.5 | 4 | G1 |
| 20. | A | U | 1.5 | 2 | B1 | 42. | A | U | 1.5 | 4 | G3 |
| 21. | B | U | 1.5 | 2 | B1 | 43. | B | U | 1.5 | 4 | G2 |
| 22. | C | H | 1.5 | 2 | B3; A1 | 44. | D | H | 1.5 | 4 | G2 |

**Multiple Choice = 66 marks**

**Part B: Written Response**

| Q | C | S | CO | PLO |
|---|---|---|---|---|
| 1. | U | 2 | 3 | F6 |
| 2. | U | 2 | 3 | F4 |
| 3. | U | 5 | 2 | C2 |
| 4. | U | 1 | 4 | G13, G8; A1 |
| 5. | H | 2 | 4 | G13, G8; A1 |
| 6. | U | 2 | 4 | G13, G8; A1 |
| 7. | U | 4 | 2 | C5 |
| 8. | U | 1 | 2 | C6 |
| 9. | H | 5 | 2 | C7, C8 |

**Written Response = 24 marks**

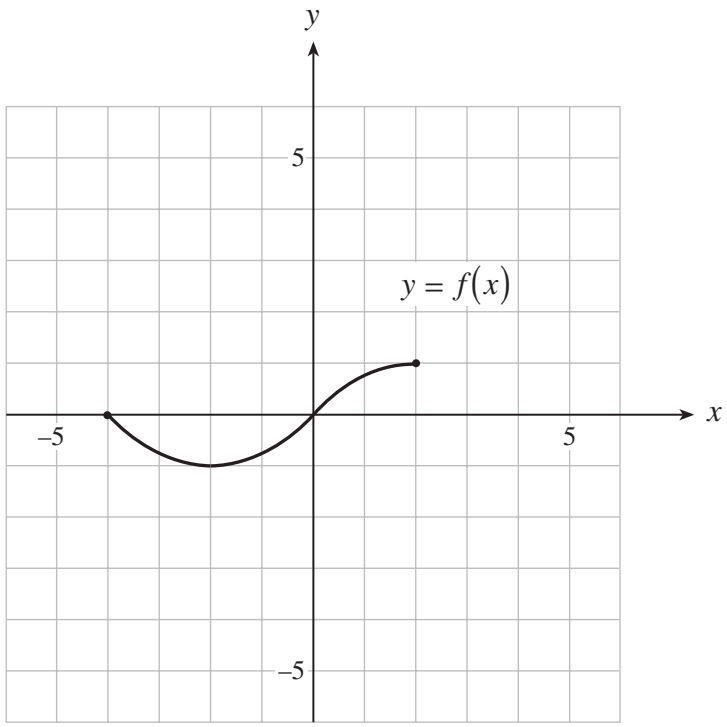
Multiple Choice = 66 (44 questions)
Written Response = 24 (9 questions)
**EXAMINATION TOTAL = 90 marks**

**LEGEND:**

| | | |
|---|---|---|
| **Q** = Question Number | **K** = Keyed Response | **C** = Cognitive Level |
| **S** = Score | **CO** = Curriculum Organizer | **PLO** = Prescribed Learning Outcome |

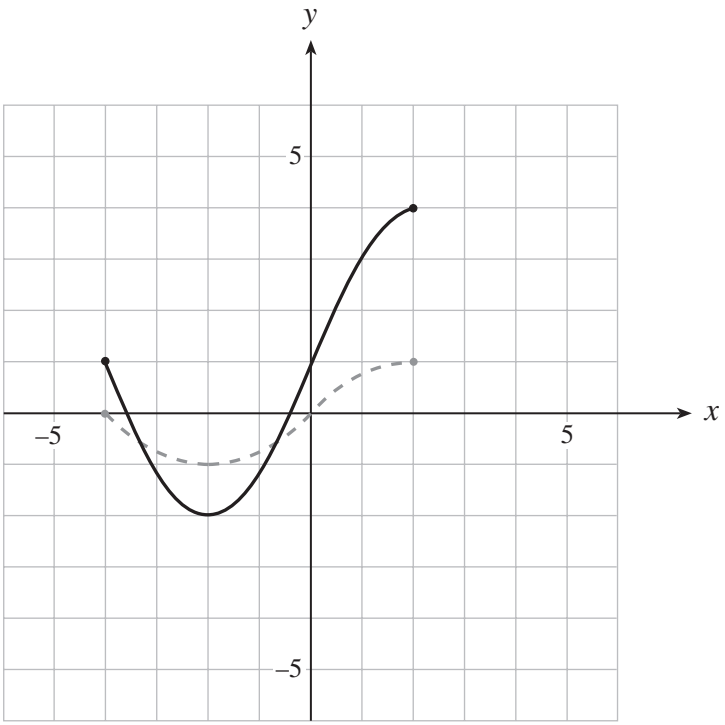
**Use the following graph to answer questions 1 and 2.**

The graph of $y = f(x)$ is shown below.



1. On the grid provided, sketch the graph of $y = 3f(x) + 1$. **(2 marks)**

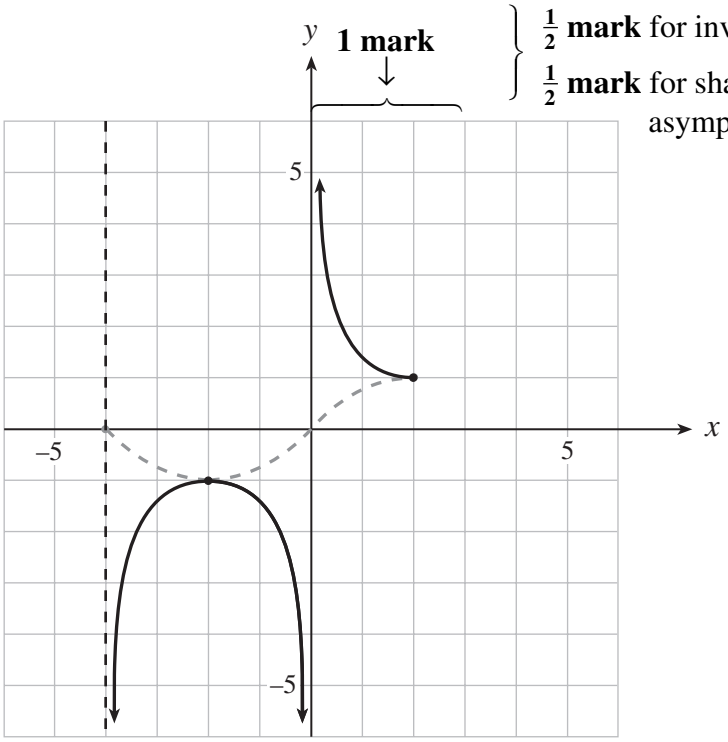
## solution




**1 mark for vertical expansion**
**1 mark for vertical translation**

2. On the grid provided, sketch the graph of $y = \dfrac{1}{f(x)}$. **(2 marks)**

**solution**




**1 mark**

$\frac{1}{2}$ **mark** for invariant point

$\frac{1}{2}$ **mark** for shape including asymptotic behaviour and truncation

**1 mark**

$\frac{1}{2}$ **mark** for invariant point

$\frac{1}{2}$ **mark** for shape – asymptotic behaviour on each end

3. Solve algebraically: $2\log_3(x+4) - \log_3(-x) = 2$ **(5 marks)**

## solution

$$\log_3\left(x^2+8x+16\right) - \log_3(-x) = 2 \qquad \leftarrow \textbf{1 mark}$$

$$\log_3 \frac{x^2+8x+16}{-x} = 2 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{x^2+8x+16}{-x} = 3^2 \qquad \leftarrow \textbf{1 mark}$$

$$x^2+8x+16 = -9x$$

$$x^2+17x+16 = 0$$

$$(x+1)(x+16) = 0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x = -1 \quad x = -16 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$\tfrac{1}{2}$ **mk** reject $\leftarrow \tfrac{1}{2}$ **mark**

## alternate solution

$$\log_3\left(x^2+8x+16\right) - \log_3(-x) = \log_3 9 \qquad \leftarrow \textbf{1}\tfrac{1}{2}\textbf{ marks}$$

$$\log_3 \frac{x^2+8x+16}{-x} = \log_3 9 \qquad \leftarrow \textbf{1 mark}$$

$$\frac{x^2+8x+16}{-x} = 9 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x^2+8x+16 = -9x$$

$$x^2+17x+16 = 0$$

$$(x+1)(x+16) = 0 \qquad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$x = -1 \quad x = -16 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$\tfrac{1}{2}$ **mk** reject $\leftarrow \tfrac{1}{2}$ **mark**

**Use the following information to answer questions 4 to 6.**

A biased (weighted) coin is designed so that the probability of a head on each flip is $\frac{3}{5}$.

4. If this biased coin is flipped 3 times, what is the probability that the first 2 flips are tails and the third flip is a head? **(1 mark)**

## solution

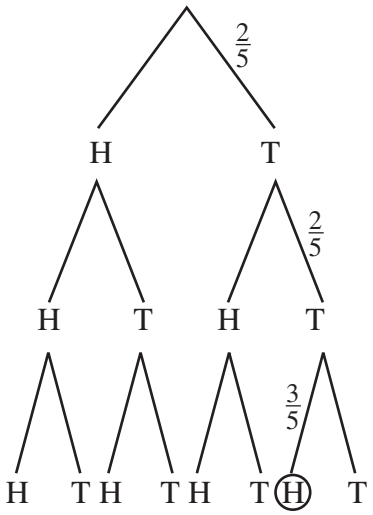
T, T, H

$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5}$ $\leftarrow$ **½ mark**

$= \frac{12}{125}$

$\approx 0.10$ $\leftarrow$ **½ mark**

## alternate solution




$\leftarrow$ **½ mark** tree diagram

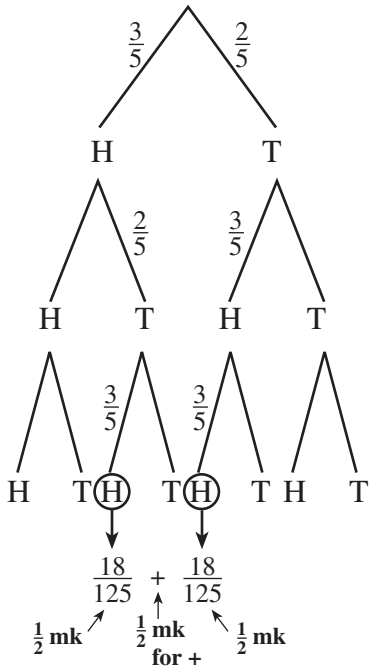
$\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{3}{5}$

$= \frac{12}{125}$

$\approx 0.10$ $\leftarrow$ **½ mark**

5. If this biased coin is flipped until exactly 2 heads appear, what is the probability that it takes exactly 3 flips until the second head appears? **(2 marks)**

## solution




$$P(\text{2H in 3 flips}) = \frac{36}{125}$$

$$\approx 0.29 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

## alternate solution

$$\overbrace{\_\ \_}^{P(\text{1H})}\ \underline{\text{H}} \qquad P(\text{H}) = \frac{3}{5}$$

$$n = 2,\ p = \frac{3}{5},\ x = 1$$

$$P(\text{2H in 3 flips}) = \overbrace{\text{binompdf}\left(2,\ \frac{3}{5},\ 1\right)}^{\textbf{1 mark}} \times \overbrace{\frac{3}{5}}^{\frac{1}{2}\textbf{ mark}}$$

$$\approx 0.29 \quad \leftarrow \tfrac{1}{2}\textbf{ mark}$$

6. If this biased coin is flipped 7 times, what is the probability that exactly 3 or 4 heads appear?
**(2 marks)**

## solution

$n = 7$

$p = \frac{1}{2}$

$x = 3,\ 4$

$$P(3\text{H}) + P(4\text{H}) = \text{binompdf}\left(7,\ \frac{3}{5},\ 3\right) + \text{binompdf}\left(7,\ \frac{3}{5},\ 4\right)$$

($\frac{1}{2}$ mark over each binompdf; $\frac{1}{2}$ mark for the $+$)

$$= 0.193536 + 0.290304$$

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

## alternate solution 1

$$\underbrace{{}_7\text{C}_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^4}_{\frac{1}{2}\text{ mark}} + \underbrace{{}_7\text{C}_3\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^3}_{\frac{1}{2}\text{ mark}}$$

($\frac{1}{2}$ mark for the $+$)

$$= 0.193536 + 0.290304$$

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

## alternate solution 2

$$\text{binomcdf}\left(7,\ \frac{3}{5},\ 4\right) - \text{binomcdf}\left(7,\ \frac{3}{5},\ 2\right)$$

($\frac{1}{2}$ mark over each binomcdf; $\frac{1}{2}$ mark for the $-$)

$$\approx 0.48 \quad \leftarrow \tfrac{1}{2} \text{ mark}$$

**Use the following equation to answer questions 7 and 8.**

$$2\cos^2 x + 3\cos x + 1 = 0$$

7. Solve the equation algebraically, giving exact values for $x$, where $0 \le x < 2\pi$. **(4 marks)**

**solution**

$$(2\cos x + 1)(\cos x + 1) = 0$$

**$\frac{1}{2}$ mark** $\rightarrow$ $\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$ $\leftarrow$ **$\frac{1}{2}$ mark**

**2 marks** $\rightarrow$ $x = \frac{2\pi}{3}, \ \frac{4\pi}{3}$ $\qquad$ $x = \pi$ $\leftarrow$ **1 mark**

8. Give the general solution for this equation.
(Solve over the set of real numbers giving exact value solutions.) **(1 mark)**

**solution**

$$\underbrace{x = \pi + 2n\pi, \ x = \frac{2\pi}{3} + 2n\pi, \ x = \frac{4\pi}{3} + 2n\pi}_{\uparrow}$$

**1 mark for** $+ 2n\pi$

$n$ is an integer

**Note: Do not deduct if *n* is an integer is missing.**

9. Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

## solution

| LEFT SIDE | RIGHT SIDE |
|---|---|
| $\cos 2x$ | $\dfrac{\cot x - \sin 2x}{\cot x}$ |
| | $\frac{1}{2}$ **mark** $\downarrow$ $\quad$ $\frac{1}{2}$ **mark** $\downarrow$ |
| | $= \dfrac{\frac{\cos x}{\sin x} - 2\sin x \cos x}{\frac{\cos x}{\sin x}}$ $\leftarrow \frac{1}{2}$ **mark** |
| | $\frac{1}{2}$ **mark** $\downarrow$ |
| | $= \dfrac{\frac{\cos x}{\sin x} - 2\sin x \cos x}{\frac{\cos x}{\sin x}}\left(\dfrac{\sin x}{\sin x}\right)$ |
| | $= \dfrac{\cos x - 2\sin^2 x \cos x}{\cos x}$ $\leftarrow$ **1 mark** |
| | $= \dfrac{\cos x\left(1 - 2\sin^2 x\right)}{\cos x}$ $\leftarrow$ **1 mark** |
| | $= 1 - 2\sin^2 x$ $\leftarrow \frac{1}{2}$ **mark** |
| | $= \cos 2x$ $\leftarrow \frac{1}{2}$ **mark** |

LS = RS

9. Prove the identity. **(5 marks)**

$$\cos 2x = \frac{\cot x - \sin 2x}{\cot x}$$

## alternate solution

| LEFT SIDE | RIGHT SIDE |
|---|---|
| $\cos 2x$ | $\dfrac{\cot x - \sin 2x}{\cot x}$ |
| | $= \dfrac{\cot x}{\cot x} - \dfrac{\sin 2x}{\cot x}$ ← **1 mark** |
| | $= 1 - \dfrac{2\sin x \cos x}{\frac{\cos x}{\sin x}}$ ← $\frac{1}{2}$ **mark** ← $\frac{1}{2}$ **mark** |
| | $\frac{1}{2}$ **mk** ↑ $\frac{1}{2}$ **mark** ↓ |
| | $= 1 - \dfrac{2\sin x \cos x}{\frac{\cos x}{\sin x}} \left(\dfrac{\sin x}{\sin x}\right)$ |
| | $= 1 - \dfrac{2\sin^2 x \cos x}{\cos x}$ ← **1 mark** |
| | $= 1 - 2\sin^2 x$ ← $\frac{1}{2}$ **mark** |
| | $= \cos 2x$ ← $\frac{1}{2}$ **mark** |

LS = RS

**END OF KEY**