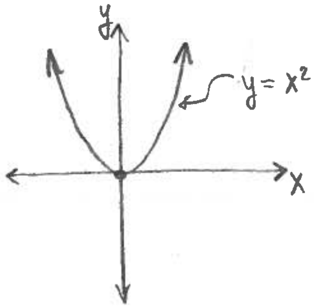
Day 1

- ☐ Introduction
- ☐ course outline + expectations

Review of Concepts necessary for Transformations

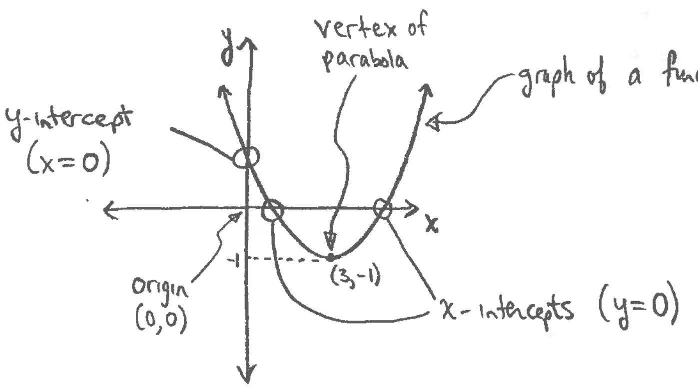
A function "f" at a point x is written as $f(x)$ and read "f at x" or "f of x". The function f maps x to only one value, $f(x)$.

Ex $f(x) = x^2$ takes pt x and maps it to the value $f(x)$, x^2 in this case
→ we often call $f(x)$ our y-value when graphing



$y = x^2$ or $y = f(x)$ where $f(x) = x^2$

Take a closer look:



* points always written as (x, y) on a Cartesian grid

Domain — the x values for which the function is defined

ex x is an real number

(OR) $x \in \mathbb{R}$ ← Set of Real Numbers
"belongs to"

Range — the y values for which the function is defined

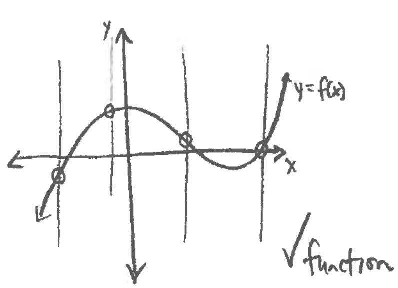
ex $x > -1$, where $x \in \mathbb{R}$

(or) $\{x \mid x > -1, x \in \mathbb{R}\}$
"such that"

Vertical Line Test

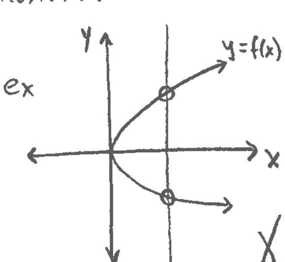
If you can draw a perfectly vertical line through 2 or more points, f is not a function. Conversely, if you can't do this, then f is a function.

ex



√ function

ex



X not a function

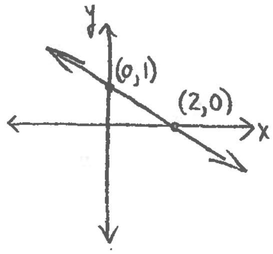
Ex. State the domain and range of each function. Find the x- and y-intercepts.

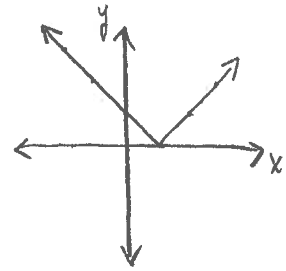
a) $x + 2y = 2$

b) $y = |x-1|$

use graphing calc. or table of values

Sol^n:





x	y
-2	$\lvert -2-1\rvert = \lvert -3\rvert = 3$
-1	$\lvert -1-1\rvert = \lvert -2\rvert = 2$
0	$\lvert 0-1\rvert = \lvert -1\rvert = 1$
1	$\lvert 1-1\rvert = \lvert 0\rvert = 0$
2	$\lvert 2-1\rvert = \lvert 1\rvert = 1$

$x=0 \Rightarrow (0) + 2y = 2$

So $y = 1$

$\therefore$ y-int is 1

$y=0 \Rightarrow x + 2(0) = 2$

So $x = 2$

$\therefore$ x-int is 2

use to graph

Domain is $\mathbb{R}$

Range is $\mathbb{R}$

$*$ always true for a line $*$

from graph:

x-int is 1 or $(0,1)$

y-int is 1 or $(1,0)$

Domain is $\mathbb{R}$ (can use any x value)

Range is $y \geq 0$ (we cannot have a negative value)

Ex. For what value(s) of x is $f(x) = \dfrac{5}{x(x+1)}$ undefined

Sol^n: denominator cannot be zero $\Longrightarrow$ $x(x+1) \neq 0$

So $x \neq 0$ and $x + 1 \neq 0$

$\Longrightarrow x \neq -1$

$\therefore$ $f(x)$ is undefined for $x = 0, -1$

$*$ this determines the domain of f $*$

Ex. If $f(x) = x^2 + 2x + 1$, determine $f(3x)$

Sol^n: $f(3x) = (3x)^2 + 2(3x) + 1$

$= 9x^2 + 6x + 1$

note: $(3x)^2$ means $(3x)(3x)$
$= 3^2 x^2$
$= 9x^2$

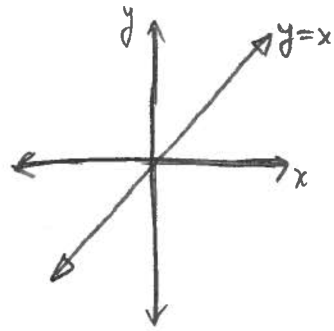
Basic Functions we'll see in this course:

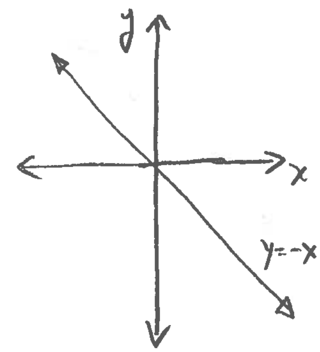
a) Linear Functions

→ form $y = mx + b$ (slope: m; y-int: b)

→ easy to plot by using slope + y-int
or by x-int + y-int (any two points defines a line)

ex

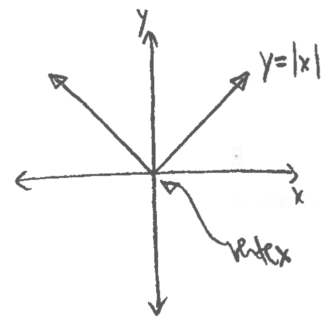




b) Absolute Value Functions

$y = |x|$ means $y = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$

→ has characteristic pointy vertex
(like two lines meeting; symmetrical)

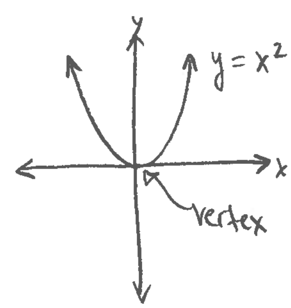


c) Quadratic Function

→ degree 2 (highest exponent on variable is 2)

→ parabolic shape

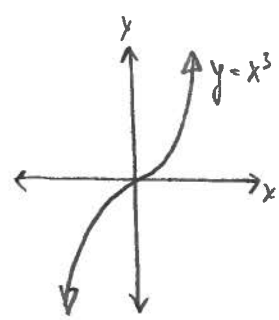
→ has vertex (max or min of function)

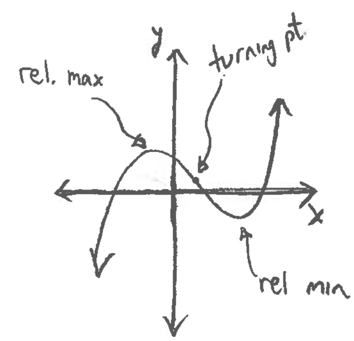


d) Cubic Function

→ degree 3

→ may have relative min + max

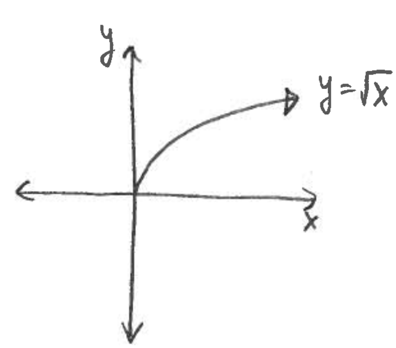




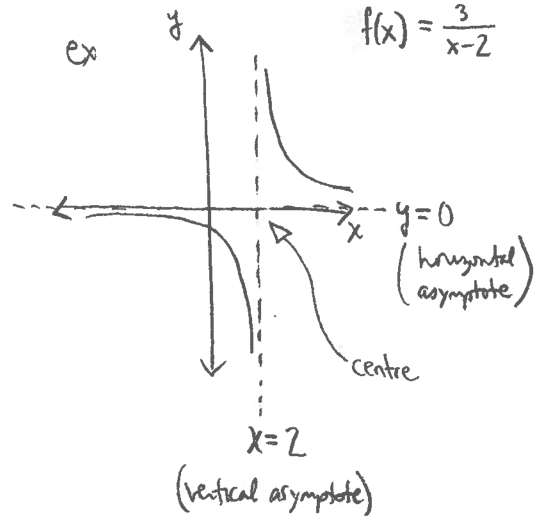
e) Radical Functions

→ ex $y = \sqrt{x}$

→ radicand (whatever is under the "$\sqrt{\ }$")
cannot be negative



Rational Function (chapter 9)



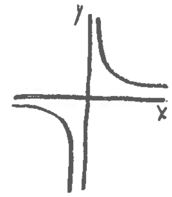
→ form $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials and $h(x) \neq 0$

→ line $y=0$ is a horizontal asymptote if $g(x) \neq 0$

→ where x is undefined, you have vertical asymptotes (b/c $h(x) \neq 0$)

PLEASE — get to know these functions & their properties
→ you should memorize their graphs! ($y=x$, $y=x^2$, $y=x^3$, $y=\sqrt{x}$, $y=|x|$)

* f) Oh! and this one: $y = \frac{1}{x}$



note $x=0$ is a v. asymptote
$y=0$ is a h asymptote

QUIZ!

Now we begin!

Unit 1: Transformations

1.1 Horizontal + Vertical Translations of Functions

Ex 1 How do the graphs of $y=|x|+2$ and $y=|x|-2$ compare to the graph of $y=|x|$?

MATH → NUM → abs(

Sol^n make table of values (OR) use graphing calculator

$y=|x|$

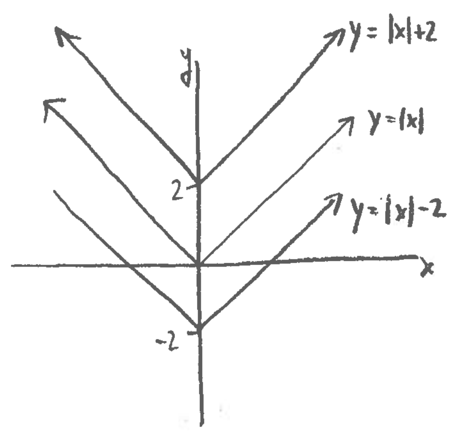
x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$y=|x|+2$

x	y
-3	5
-2	4
-1	3
0	2
1	3
2	4
3	5

$y=|x|-2$

x	y
-3	1
-2	0
-1	-1
0	-2
1	-1
2	0
3	1

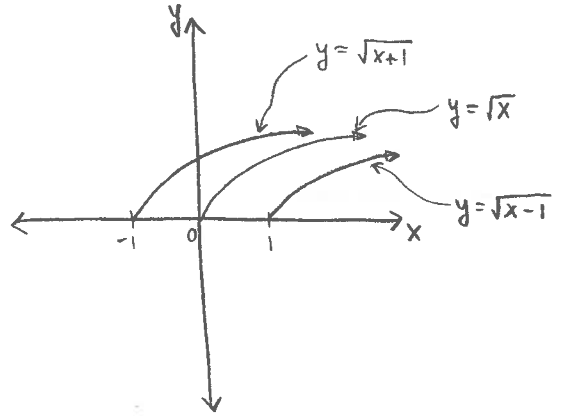


Notice that $y = |x| + 2$ is the graph of $y = |x|$ translated UP 2 units
and that $y = |x| - 2$ is the graph of $y = |x|$ translated DOWN 2 units

In general, the graph $y = f(x) + k$ (or $y - k = f(x)$) is the graph of $y = f(x)$ translated UP by k units if $k > 0$, or it is translated DOWN by k units if $k < 0$.

Ex2 How do the graphs of $y = \sqrt{x+1}$, $y = \sqrt{x-1}$, and $y = \sqrt{x}$ compare?

Sol^n:



use graphing calculator

OR

table of values

$y = \sqrt{x+1}$

x	y
-1	0
0	1
3	2

$y = \sqrt{x}$

x	y
0	0
1	1
4	2

$y = \sqrt{x-1}$

x	y
1	0
2	1
5	2

Notice that $y = \sqrt{x+1}$ is the graph of $y = \sqrt{x}$ translated 1 unit LEFT
and $y = \sqrt{x-1}$ is the graph of $y = \sqrt{x}$ translated 1 unit RIGHT

In general, the graph $y = f(x-h)$ is the graph of:

$y = f(x)$ translated h units to the RIGHT if $h > 0$,
or it is translated h units to the LEFT if $h < 0$.

watch out! Signs can be confusing!!

ex $y = (x - 5)^2$

⊖ means right

So $y = (x-5)^2$ same as $y = x^2$, but translated right 5 units

$y = |x + 1|$

⊕ means left

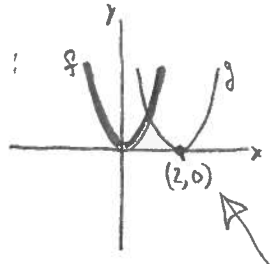
So $y = |x+1|$ same as $y = |x|$ but translated left 1 unit

Ahhh!! Can't remember!
(or hate to memorize) } use a test point to see if it's left or right:

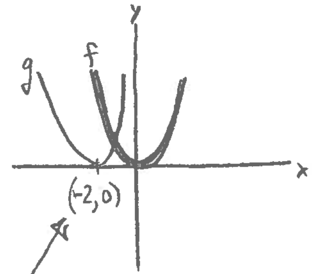
Ex graphing: $g(x) = (x+2)^2$ from $f(x) = x^2$

you can think this in your head until it becomes engrained ☺

You know it's Either:



or:



not here!

Try x-value in g: $g(2) = (2+2)^2$
$= 4^2$
$= 16$ ← y value

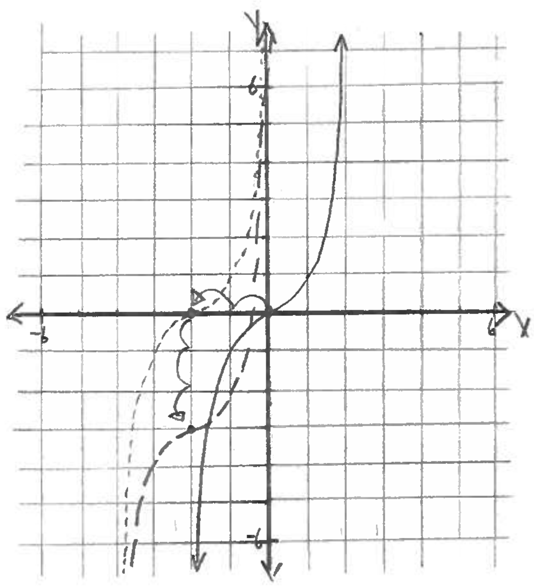
So $(2,16)$ is a point of g

yes! $g(-2) = (-2+2)^2$
$= 0^2$
$= 0$ ← y value

So $(-2,0)$ is point of g

Ex 3 Sketch graph of $y = (x+2)^3 - 3$

Sol^n * order doesn't matter here: can do either horizontal or vertical translation 1st *



— First graph base function $y = x^3$

— then translate <u>left</u> 2

— then translate result <u>down</u> 3

$y = (x \oplus 2)^3 \ominus 3$

left ($\ominus$ is right)

down ($\oplus$ is up)

—— $y = x^3$

------ $y = (x+2)^3$

– – – $y = (x+2)^3 - 3$

tips:

- (h, k) will be the centre/vertex for $y = f(x-h) + k$ $h, k > 0$
- can do the two translations together once you're comfortable
- can do table of values too (but not as efficient)
- can move key points, then connect (see next example)

Ex 4 If $(3,-4)$ is a point on the graph of $y = f(x)$, determine a point on the graph of $y = f(x-2)-1$.

great quiz/test question !! :)

Sol^n: $y = f(x - 2) - 1$

moves x-value right by 2: $x = 3 \rightarrow x = 5$

moves y-value down by 1: $y = -4 \rightarrow y = -5$

using mapping notation: $(x, y) \rightarrow (x+2, y-1)$

Thus, $(5, -5)$ is a point on the graph of $y = f(x-2)-1$.

Homework:

- ☐ Checkout Course website: http://precalc12.yolasite.com
- ☐ Study for Quiz: know how to quickly sketch basic functions
- ☐ Textbook: pg 12-15 #1, 4, 5-11, 19, C3, C4