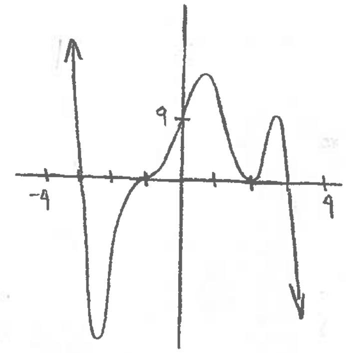
Day 10 Review

① Determine a polynomial equation for the graph of function to the right.

② Without technology, sketch the graph of $g(x) = (x^3+3x^2-4)(x^2+6x+9)$



---

sol^n: ① x int: $-3 \quad -1 \quad 2 \quad 3$

$\therefore$ factors: $(x+3) \quad (x+1) \quad (x-2) \quad (x-3)$

(minimum) Multiplicity: $1 \quad 3 \quad 2 \quad 1$ $\Rightarrow$ deg 7 & we see graph goes from quad II $\to$ IV

$\therefore$ leading coefficient must be negative

Possible equation of form $y = a(x+3)(x+1)^3(x-2)^2(x-3)$, where $a<0$

Use y-int to find a, i.e., sub $(0,9)$ into eqn: $9 = a(0+3)(0+1)^3(0-2)^2(0-3)$

$9 = a(-36)$

$\therefore a = \frac{-9}{36} = \frac{-1}{4}$

Therefore eqn could be $y = -\frac{1}{4}(x+3)(x+1)^3(x-2)^2(x-3)$

---

② First job is to factor as much as possible:

$g(x) = (x^3+3x^2-4)(x+3)^2$

let $P(x) = x^3+3x^2-4$ possible zeros: $\pm 1, \pm 2, \pm 4$

try $a=1 \Rightarrow P(1) = (1)^3+3(1)^2-4$

$= 0$

$\therefore x-1$ is a factor of $P(x)$

Divide $P(x)$ by $(x-1)$ ...

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 0 & -4 \\ & & -1 & -4 & -4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$\therefore x^2+4x+4$ is a factor of $P(x)$

$\therefore g(x) = (x^2+4x+4)(x-1)(x+3)^2$

$= (x+2)^2(x-1)(x+3)^2$

analyze $g(x) = (x+2)^2(x-1)(x+3)^2$

zeros: $-2 \quad 1 \quad -3$

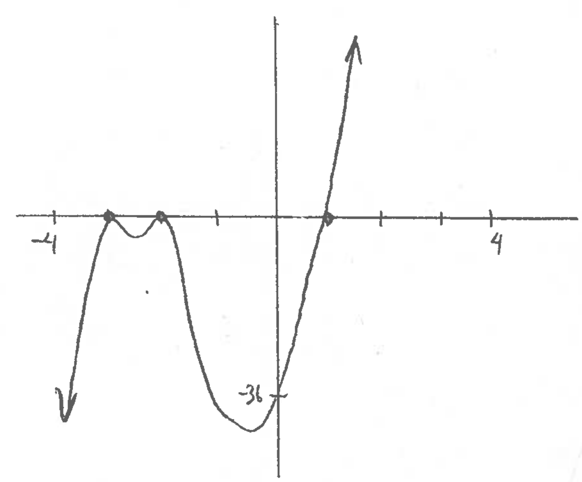
multiplicity: $2 \quad 1 \quad 2$

$\therefore$ deg is 5 and ⊕ leading co-efficient

$\Longrightarrow$ graph goes from quad III $\to$ I

y-int (set $x=0$): $g(0) = (0+2)^2(0-1)(0+3)^2 = -36$

Putting it all together....



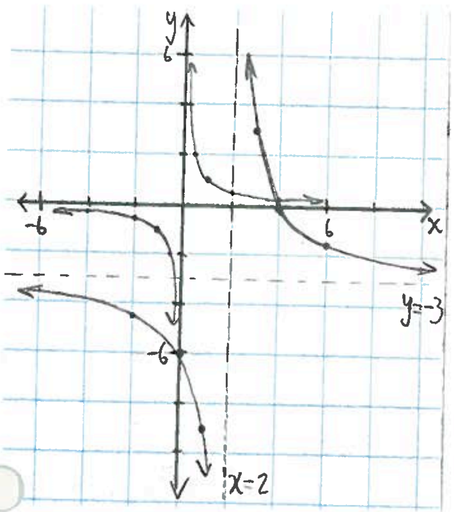
CHAPTER 9: Rational Functions

9.1 Exploring Rational Functions Using Transformations

A rational function is a fn that can be written as $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ & $q(x)$ are polynomials and $q(x) \neq 0$ Ex. $y = \frac{20}{x}$, $C(n) = \frac{100+2n}{n}$, $f(x) = \frac{3x^2-4}{x-5}$

Recall: We are already familiar with Fns of the form $y = \frac{a}{x-h} + k$ as being transformed from $y = \frac{1}{x}$.

Ex1 Sketch $y = \frac{6}{x-2} - 3$ and identify any asymptotes. State Domain & Range.



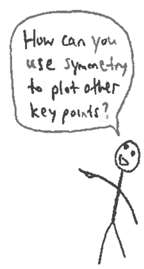
MAPPING NOTATION

$y = \frac{1}{x} \longrightarrow y = \frac{6}{x-2} - 3$

$(x, y) \longrightarrow (x+2, 6y-3)$

x	y
-4	$-\frac{1}{4}$
-2	$-\frac{1}{2}$
-1	-1
1	1
2	$\frac{1}{2}$
4	$\frac{1}{4}$

x	y
$(-4)+2=$ -2	$-4.5 = 6(\frac{-1}{4}) - 3$
0	-6
1	-9
3	3
4	0
6	-1.5



Notice vertical asymptote at $x=2$ and horizontal asymptote at $y=-3$ read directly from eqn ☺

$y = \frac{6}{x-2} - 3$

Use this for domain & range:

D: $\{x \mid x \neq 2, x \in \mathbb{R}\}$

R: $\{y \mid y \neq -3, y \in \mathbb{R}\}$

Ex2 Graph $y = \frac{4x-5}{x-2}$. Identify any asymptotes and intercepts.

y-int. let $x=0$:

$y = \frac{4(0)-5}{(0)-2} = \frac{5}{2}$ $\therefore (0, 2.5)$

x-int let $y=0$:

$0 = \frac{4x-5}{x-2}$

$\therefore 4x-5=0$

$x = \frac{5}{4}$ $\therefore (1.25, 0)$

Put in standard form to see transformations from $y = \frac{1}{x}$

$y = \frac{4x-5}{x-2}$

$= \frac{4x-8+8-5}{x-2}$

Add "0" to get $(x-2)$ factor

$= \frac{(4x-8)+(8-5)}{x-2}$

$= \frac{4(x-2)+3}{x-2}$

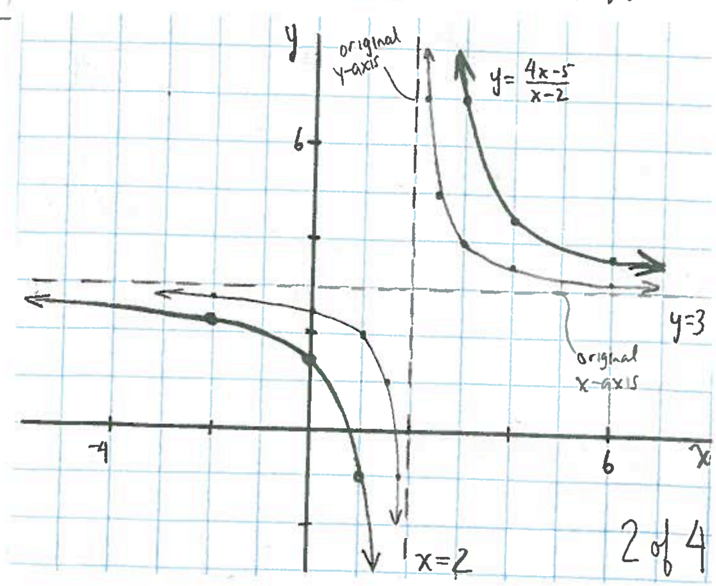
$= \frac{4(x-2)}{x-2} + \frac{3}{x-2}$

$= 4 + \frac{3}{x-2}$

$\therefore y = 3\left(\frac{1}{x-2}\right) + 4$

V. stretch of x3 right 2 up 4

* Here's a quick & clever way to graph $y = \frac{4x-5}{x-2}$ from $y = \frac{1}{x}$ by TRANSLATING the axes instead of the graph of $y = \frac{1}{x}$!



Notes on graph to right

- original axes became asymptotes for $y = \frac{4x-5}{x-2}$
- performed vertical stretch of factor 3 before translating axes 2 left & 3 down

axes translated in OPPOSITE way so graph of $y = \frac{4x-5}{x-2}$ will have centre (2, 4)

Ex 3 Cell phone plan A: \$10 per month plus 10¢ per text or minute of talk time.
Cell phone plan B: \$5 per month plus 15¢ per text or minute of talk time.

a) Represent the average cost per text or min of each plan with a rational FN.

b) Use the graphs of these functions to decide which is the better plan.

Sol'n:

a) Let x be the combined number of texts & minutes and $A(x)$ & $B(x)$ are average cost (in \$) of plan A & B, respectively.

$$\text{Average Cost} = \frac{\text{total cost}}{\text{Combined number of texts + mins.}}$$

Total Cost (plan A) = \$10 + \$0.10 x

Total Cost (plan B) = \$5 + \$0.15 x

Therefore $A(x) = \dfrac{10 + 0.1x}{x}$ & $B(x) = \dfrac{5 + 0.15x}{x}$

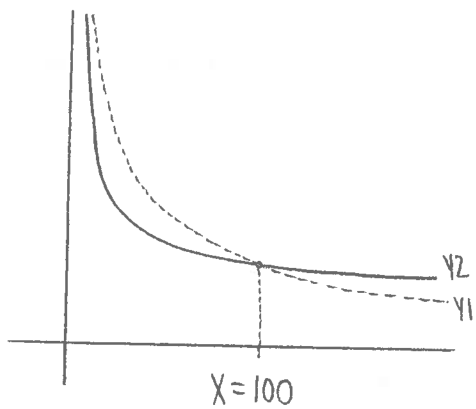
b) Use graphing calc:

$Y1 = (10 + 0.1X)/X$

$Y2 = (5 + 0.15X)/X$

Possible window:

	Min	max
X	0	300
Y	0	1



Use $\boxed{\text{INTERSECT}}$ FN to find x value where both plans are of equal value

We see this is at 100 min & texts.

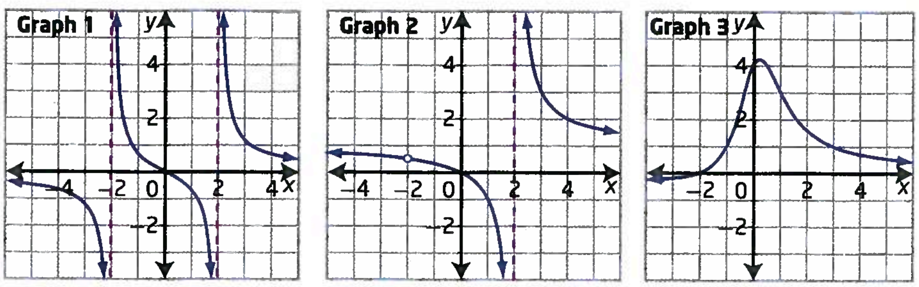
So plan A is better if your combined min or texts is more than 100/month and plan B is better if it's less.

HWR pg 442-445 #(1-4)d, 5, 7d, 8, 12, 16

9.2 Analysing Rational FNS

Points of discontinuity vs asymptotes

Ex 1 Match the equation of each rational function with the most appropriate graph. Explain how you made your choice.



$A(x) = \dfrac{x^2+2x}{x^2-4}$

$B(x) = \dfrac{2x+4}{x^2+1}$

$C(x) = \dfrac{2x}{x^2-4}$

Sol^n Factoring is key! Then look for non-permissible values (x values that are undefined)

$A(x) = \dfrac{x\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$ } factor of both numerator + denominator correspond to point of discontinuity

$\longrightarrow x \neq -2$ but otherwise the factor has divided out + doesn't affect look of graph

$\longrightarrow A(-2) = \frac{-2}{-2-2} = \frac{1}{2}$ $\therefore$ pt of discontinuity at $(-2, \frac{1}{2})$

Behaviour near pt of discontinuity: as $x \to -2$, $y \to \frac{1}{2}$

means "approaches"

$= \dfrac{x}{x-2}$

factor only in denominator corresponds to a vertical asymptote

$\longrightarrow x-2 \neq 0 \Rightarrow x \neq 2$ + so $x = 2$ is an asymptote

factor(s) of numerator correspond to x-intercept(s)

$\longrightarrow$ set numerator $= 0 \Rightarrow x = 0$ is an x-int. $\therefore$ graph 2

$B(x) = \dfrac{2x+4}{x^2+1}$ $\longrightarrow$ let $x=0 \Rightarrow B(0) = \frac{2(0)+4}{(0)^2+1} = 4$ $\therefore$ y-int is 4

$= \dfrac{2(x+2)}{x^2+1}$

$\longrightarrow$ set $2(x+2) = 0 \Rightarrow x = -2$ is an x-intercept

$\longrightarrow$ no factors in common in num + dem $\Rightarrow$ no pt of discontinuity.

$\longrightarrow$ $x^2+1 \neq 0$ for any value of $x \Rightarrow$ no vertical asymptote $\therefore$ graph 3

$C(x) = \dfrac{2x}{x^2-4}$ $\longrightarrow$ let $x=0 \Rightarrow y=0$ is intercept

$= \dfrac{2x}{(x-2)(x+2)}$

$\longrightarrow$ $x = 0$ is an x-intercept (can see from above step)

$\longrightarrow$ vertical asymptote at $x = 2$ + $x = -2$

No factors in common $\Rightarrow$ no pts of discontinuity (for num + den)

$\therefore$ graph 1

Behaviour near vertical asymptote: as $x \to -2$ or 2, $y \to \pm\infty$

HWK p 452-456 # 2, 4-8 challenge: 20, 21, 22, 23 (optional ☺)