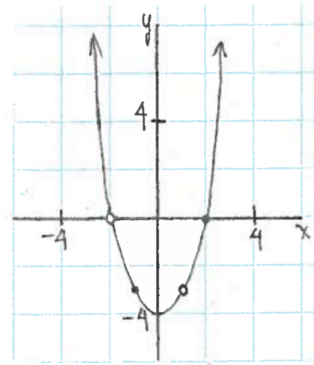
Day 11 Review

① pg 443 #11: Write $y = \dfrac{x-2}{2x+4}$ in form $y = \dfrac{a}{x-h} + k$ and sketch its graph using transformations

② pg 455 #21 b): Write the equation of the rational function shown to the right. Leave your answer in factored form.



Sol'n ① Factor denominator and manipulate numerator to get same factor in numerator by "adding zero":

$$y = \frac{x-2}{2(x+2)}$$ want this in numerator

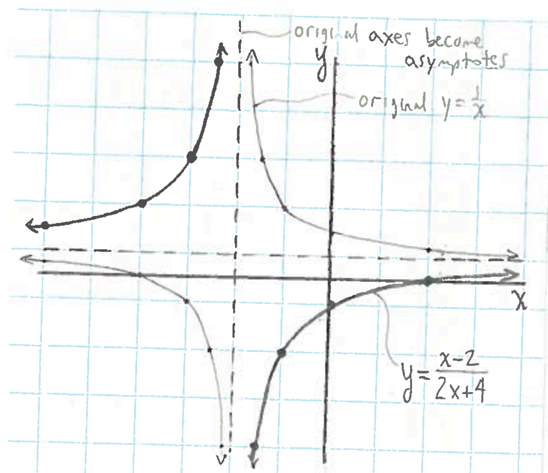
$$= \frac{x \boxed{+2-2} -2}{2(x+2)}$$ add zero

$$= \frac{x+2-4}{2(x+2)}$$

$$= \frac{x+2}{2(x+2)} - \frac{4}{2(x+2)}$$

$$= \frac{1}{2} - \frac{2}{x+2}$$

$$\therefore y = \frac{-2}{x+2} + \frac{1}{2}$$



② $x = -2$ & $x = 1$ are points of discontinuity, therefore $(x+2)$ & $(x-1)$ are factors of both the numerator and the denominator. They do not affect overall shape of graph.

So far: $y = \dfrac{(x+2)(x-1)}{(x+2)(x-1)}$

There are no vertical asymptotes, so no other factors in the denominator. The graph (aside from discontinuities) is $x^2 - 4$; thus, we must have:

$$y = \frac{(x^2-4)(x+2)(x-1)}{(x+2)(x-1)}$$

$$= \frac{(x+2)(x-2)(x+2)(x-1)}{(x+2)(x-1)}$$

$$\therefore y = \frac{(x+2)^2(x-1)(x-2)}{(x-1)(x+2)}$$

9.3 Connecting Graphs & Rational Equations

Similar methods we looked at for solving radical functions: algebraically or graphically

- algebra yields exact solutions but check for extraneous roots
- graphing yields solutions that may be approximate but no extraneous roots

Ex1: Solve $\frac{x}{2x+5} + 2x = \frac{8x+15}{4x+10}$ algebraically & graphically

Sol^n: algebraically

$$\frac{x}{2x+5} + 2x = \frac{8x+15}{2(2x+5)}$$

$$\cancel{2(2x+5)}\frac{x}{\cancel{2x+5}} + 2(2x+5)(2x) = \frac{8x+15}{\cancel{2(2x+5)}}\cdot \cancel{2(2x+5)}$$

$$2x + 8x^2 + 20x = 8x + 15$$

$$8x^2 + 14x - 15 = 0$$

$(8)(-15) = -120$
try factors of -120:
$20 - 6 = 14$ ✓
$(20)(-6) = -120$ ✓

$$8x^2 - 6x + 20x - 15 = 0$$

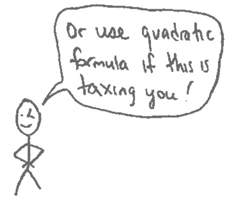
$$2x(4x-3) + 5(4x-3) = 0$$

$$(4x-3)(2x+5) = 0$$

$\therefore$ $4x - 3 = 0$ or $2x + 5 = 0$

$4x = 3$ $\quad$ $2x = -5$

$x = \frac{3}{4}$ $\quad$ $x = \frac{-5}{2}$



check extraneous roots!

$x = \frac{-5}{2}$ is extraneous b/c you can't have zero in denominator

$2(\frac{-5}{2}) + 5 = 0$ ✗ reject!

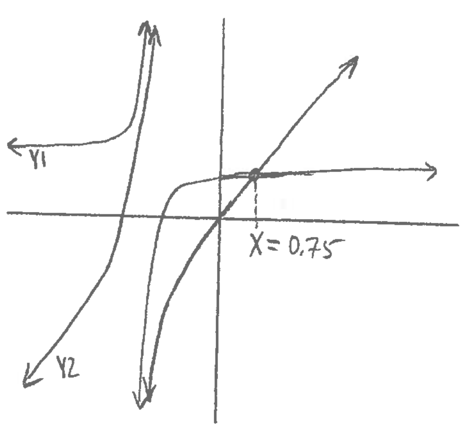
$\therefore$ Solution is $x = \frac{3}{4}$

graphically

method i) $Y1 = X/(2X+5) + 2X$

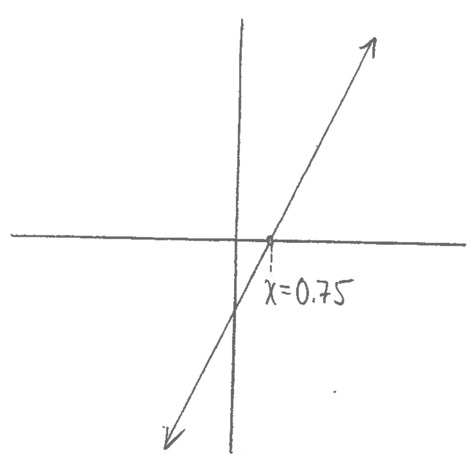
$Y2 = (8X+15)/(4X+10)$

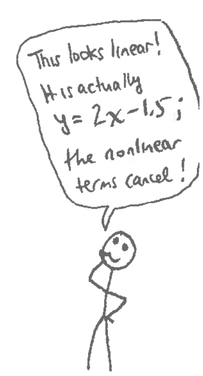
use $\boxed{\text{Intersect}}$ function



method ii) $Y1 = X/(2X+5) + 2X - (8X+15)/(4X+10)$

use $\boxed{\text{ZERO}}$ function





HWK pg 465-467 # 2, 6-9, C2

Chapter 10: Function Operations

10.1 Sums & Differences of FNS

You can add two functions, $f(x)$ and $g(x)$, to form the combined function $(f+g)(x)$.
i.e., $h(x) = f(x) + g(x)$ & we write it as $h(x) = (f+g)(x)$

Ex 1 $f(x) = 2x+1$ & $g(x) = x^2$

a) determine equation for $h(x) = (f+g)(x)$
b) sketch graphs of $f(x)$, $g(x)$, $h(x)$ on same axes
c) State Domain + Range of $h(x)$
d) Determine values of $f(x)$, $g(x)$, $h(x)$ when $x=4$

soln: a) $h(x) = (f+g)(x)$
$= f(x) + g(x)$
$= 2x+1 + x^2$
$\therefore h(x) = x^2 + 2x + 1$

c) $D = \{x \mid x \in \mathbb{R}\}$ ← common domain between f+g

$R = \{y \mid y \geq 0, y \in \mathbb{R}\}$ ← check graph

d) $f(4) = 2(4)+1$
$= 8+1$
$= 9$

$g(4) = (4)^2$
$= 16$

$h(4) = f(4) + g(4)$
$= 9 + 16$
$= 25$

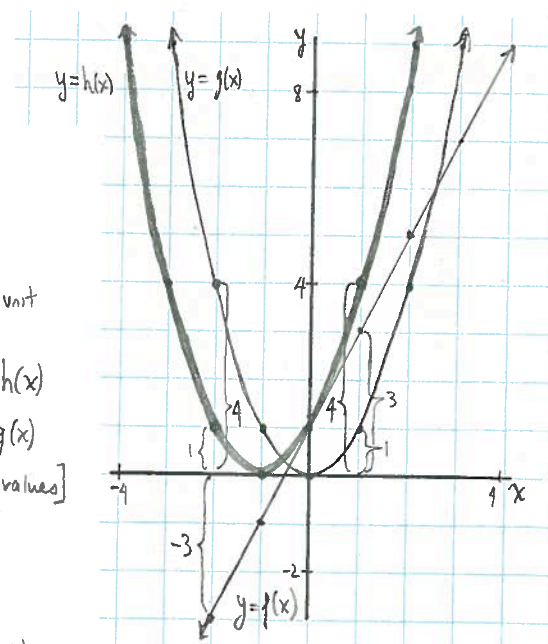
check: $h(4) = (4)^2 + 2(4) + 1$
$= 16 + 8 + 1$
$= 25$ ✓

b) Use table of values or your knowledge of transformations + lines to plot:

$f(x) = 2x + 1$ (slope: 2, y-int: 1)

$g(x) = x^2$ (base fn!)

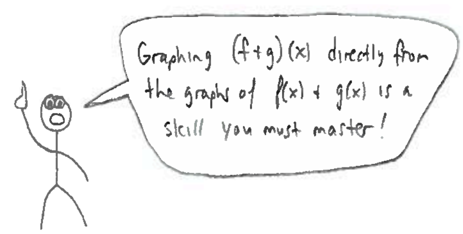
$h(x) = x^2 + 2x + 1$
$= (x+1)^2$ translate left 1 unit



☆ NOTICE: we can graph h(x) by adding values of f(x) + g(x) directly from graph! [y-values]

ex $h(1) = f(1) + g(1)$
$= 3 + 1$
$= 4$ (see graph)

ex $h(-2) = f(-2) + g(-2)$
$= -3 + 4$
$= 1$



You can subtract two functions, $f(x)$ and $g(x)$, to form combined function $(f-g)(x)$.
i.e., $h(x) = f(x) - g(x)$ & we write it as $h(x) = (f-g)(x)$.

Ex 2 $f(x) = \sqrt{x-1}$ & $g(x) = x-2$

a) Write eqn of $h(x) = (f-g)(x)$

b) Sketch graphs of $f(x)$, $g(x)$, $h(x)$ on same axes

c) Find approx domain & range (use graphing calc) of $h(x)$

Soln: a) $h(x) = (f-g)(x)$

$= f(x) - g(x)$

$= \sqrt{x-1} - (x-2)$

$\therefore h(x) = \sqrt{x-1} - x + 2$

b) • Use knowledge of radical FNS & lines to sketch $f(x)$ & $g(x)$ — or table of values if you must!

$f(x) = \sqrt{x-1}$

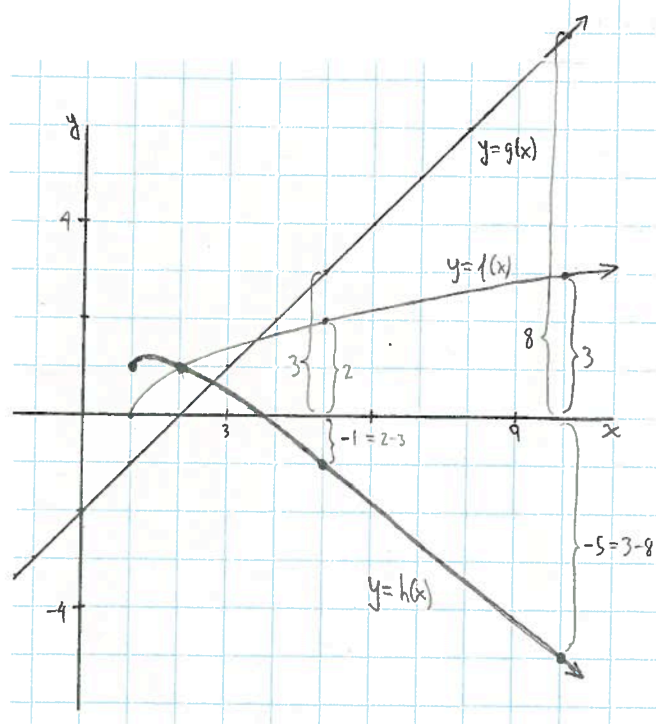
base FN $y=\sqrt{x}$ translated 1 right

$g(x) = x-2$

base FN $y=x$ translated 2 down

• Then subtract y-values: $f(x) - g(x)$ to get values of $h(x)$ for each x you choose ☺

(you may also use table of values)



c) domain is easy:

only what's common to both f&g → $D = \{x \mid x \geq 1, x \in \mathbb{R}\}$

For range, use graphing calc to find max!

(use $\boxed{\text{MAXIMUM}}$ function for $Y1 = \sqrt{(X-1)} - X + 2$)

max is 1.25 at x = 1.25

depends on graph. (can use graphing calc.) → $\therefore R = \{y \mid y \leq 1.25, y \in \mathbb{R}\}$

★ Closer look at domain of $h(x)$:

domain of $f(x)$ is $x \geq 1, x \in \mathbb{R}$

domain of $g(x)$ is $x \in \mathbb{R}$

What is the domain in common (i.e., that satisfies both FNs)

$x \geq 1$, $x \in \mathbb{R}$

$\therefore$ domain of $h(x)$ is $x \geq 1$, $x \in \mathbb{R}$

HWK: p483-485 #3, 6-8, 10, 13