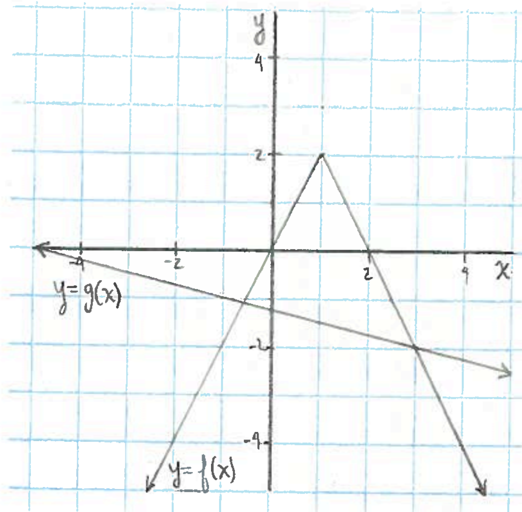
Day 12: Review

① p 467 # 13 Rachel scored 2 goals of 28 shots on net so far this year. Her goal is for a 30% shooting percentage. If from now on Rachel scores half of her shots on net, how many more shots will it take for her to reach her goal?

② Given the graphs of $y=f(x)$ and $y=g(x)$ shown to the right, sketch the graphs of:

a) $y=(f+g)(x)$

b) $y=(f-g)(x)$



Sol'n: ① Rachel's current percentage is $\frac{2}{28} \approx 7\%$

let x be the number of new shots on net, and $P(x)$ be her new percentage based on x.

If she scores $\frac{1}{2}$ her shots on net, she'll score $0.5x$.

Her new percentage will be $P(x) = \frac{2+0.5x}{28+x}$

We want $P(x)=0.3$ to get: $0.3 = \frac{2+0.5x}{28+x}$

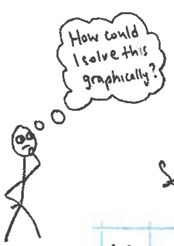
Solve for x: $(28+x)(0.3) = \frac{(2+0.5x)\cancel{(28+x)}}{\cancel{28+x}}$

$$8.4+0.3x = 2+0.5x$$

$$6.4 = 0.2x$$

$$\therefore x = 32$$

So it'll take her 32 more shots to get 30%.



② method 1: use FN knowledge: $f(x) = -2|x-1|+2$

$g(x) = \frac{-1}{4}x - \frac{5}{4}$

$\therefore (f+g)(x) = -2|x-1| - \frac{x}{4} + \frac{3}{4}$

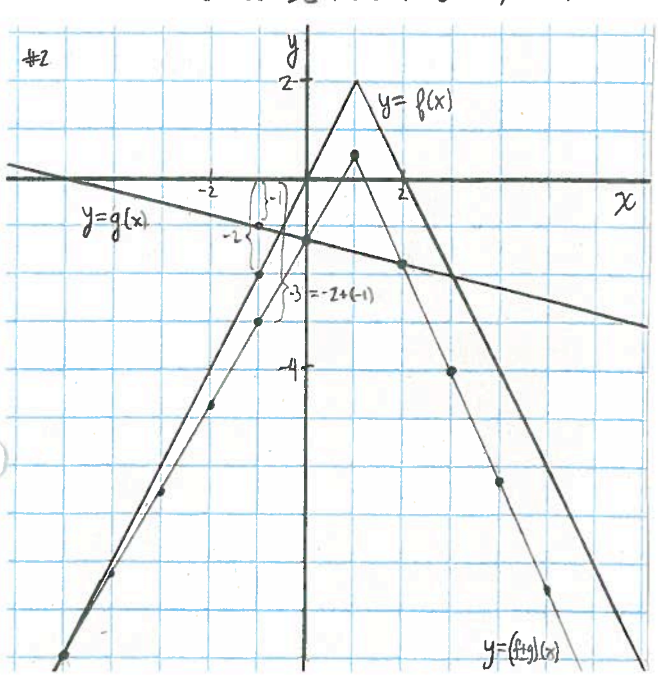
and $(f-g)(x) = -2|x-1| + \frac{x}{4} + \frac{13}{4}$

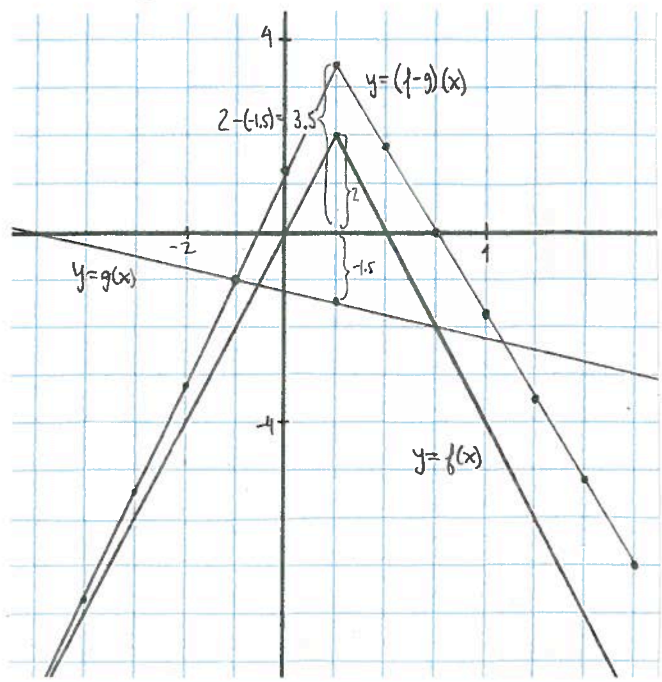
You'd probably want a graphing calc!

... or use table of values (slow)

So why not use method 2...

method 2: Add & subtract y-values directly from graph!





10.2 Products & Quotients of Functions

You can multiply two functions, $f(x)$ and $g(x)$, to form the combined function $(f \cdot g)(x)$.
i.e., $h(x) = f(x)\,g(x)$ can be written as $h(x) = (f \cdot g)(x)$.

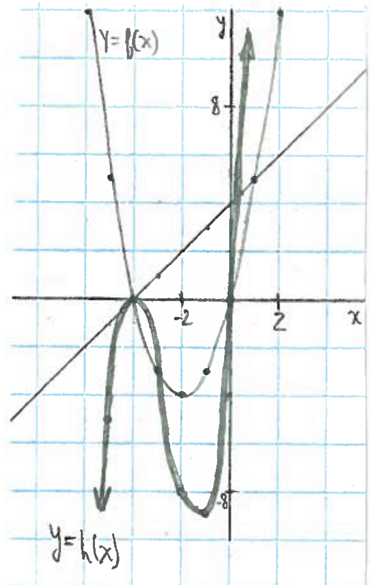
Ex 1 Given $f(x) = (x+2)^2 - 4$ and $g(x) = x + 4$, determine $h(x) = (f \cdot g)(x)$.
Graph $y = h(x)$; then state the domain + range of $h(x)$.

Soln:
$$\begin{aligned} h(x) &= (f \cdot g)(x) \\ &= f(x)\,g(x) \\ &= \left[(x+2)^2 - 4\right]\left[x+4\right] \\ &= (x^2 + 4x + 4 - 4)(x+4) \\ &= (x^2 + 4x)(x+4) \end{aligned}$$

$\therefore h(x) = x(x+4)^2$ ← use this to graph $y = h(x)$

So $h(x) = x(x^2 + 8x + 16)$

$$\boxed{h(x) = x^3 + 8x^2 + 16x}$$



Now Domain of $f(x)$ is $x \in \mathbb{R}$ + Domain of $g(x)$ is $x \in \mathbb{R}$
$\Longrightarrow$ domain of $h(x)$ will be what they both have in common $\therefore x \in \mathbb{R}$ as well.

For Range, we need to examine the graph and it's clear that $y \in \mathbb{R}$ too.

* Notice that you can sketch the graph of $h(x)$ by multiplying y-values for $f(x)$ + $g(x)$ for each x. Verify this for $x = -5, -4, -3, -2, -1, 0$ above ☺

HWK: p 496 # 2–5, (6–8) d

You can divide two functions, $f(x)$ and $g(x)$, to form the combined function $\left(\frac{f}{g}\right)(x)$.

i.e., $h(x) = \dfrac{f(x)}{g(x)}$ can be written as $h(x) = \left(\frac{f}{g}\right)(x)$.

<u>Ex 2</u> $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

a) Write simplified equation for $h(x) = \left(\frac{g}{f}\right)(x)$.

b) Sketch the graphs of $f(x)$, $g(x)$, $h(x)$ on same axes.

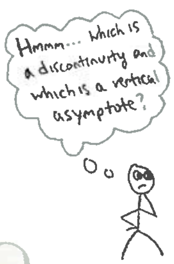
c) State domain & range of $h(x)$.

Sol'n: a) $h(x) = \left(\frac{g}{f}\right)(x)$

$$= \frac{g(x)}{f(x)}$$

$$= \frac{2x+6}{x^2+x-6}$$

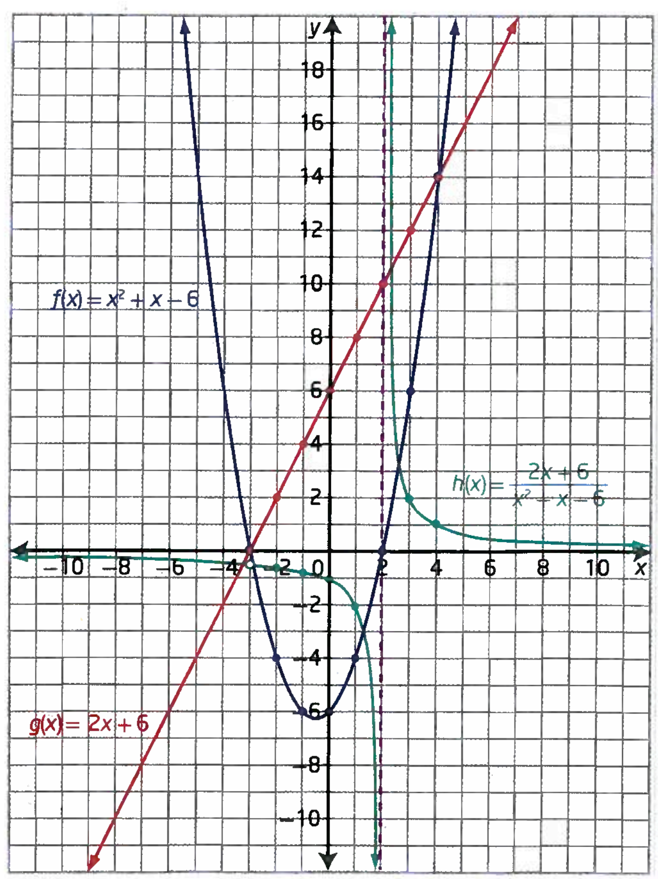
$$= \frac{2(x+3)}{(x-2)(x+3)}$$ ← we can see $x \neq 2, -3$



$\therefore h(x) = \dfrac{2}{x-2}$, $x \neq -3, 2$

b) Use a graphing calculator to graph or table of values:

x	$f(x) = x^2+x-6$	$g(x) = 2x+6$	$h(x) = \frac{2}{x-2}$, $x \neq -3, 2$
-3	0	0	DNE
-2	-4	2	$-\frac{1}{2}$
-1	-6	4	$-\frac{2}{3}$
0	-6	6	-1
1	-4	8	-2
2	0	10	DNE
3	6	12	2
4	14	14	1



c) domain of $f(x)$ is $x \in \mathbb{R}$

domain of $g(x)$ is $x \in \mathbb{R}$

domain of $h(x)$ is what's common to both $f(x)$ & $g(x)$

<u>BUT</u> also must further restrict x-values that make numerator zero i.e., where $g(x) = 0$

$\therefore$ domain of $h(x)$ is $\{x \mid x \neq -3, 2;\ x \in \mathbb{R}\}$

Range you can get from the graph (or from the eqn of $h(x)$ with your rational Fn knowledge):

discontinuity @ $x = -3 \Rightarrow y \neq \frac{2}{(-3)-2} = -\frac{2}{5}$

horizontal asymptote @ $y = 0$ (numerator can't be zero) (since $2 \neq 0$)

$\therefore$ range of $h(x)$ is $\{y \mid y \neq 0, -\frac{2}{5};\ y \in \mathbb{R}\}$

10.3 Composite Functions

The composition of $f(x)$ & $g(x)$ is defined as $f(g(x))$ (also written as $(f \circ g)(x)$) and is read as "f of g of x" or "f at g of x" or "f composed with g". $(f \circ g)(x) \neq (f \cdot g)(x)$

open circle

* $f(g(x))$ only exists for x values in domain of g so that $g(x)$ is in domain of f.

Ex 1: If $f(x) = 4x$, $g(x) = x+6$, determine:

a) $f(g(3))$ b) $g(g(3))$

visual:

$$\begin{tikzcd} x \arrow[r, "g(x)"] \arrow[rr, bend left, "f(g(x))"] & x+6 \arrow[r, "f(x)"] & 4(x+6) \end{tikzcd}$$

Soln: a) method 1:

$g(3) = (3)+6 = 9$

$\therefore f(g(3)) = f(9) = 4(9) = 36$

method 2:

$f(g(x)) = f(x+6) = 4(x+6) = 4x+24$

$\therefore f(g(3)) = 4(3)+24 = 36$

b) $g(g(x)) = g(x+6)$

$= (x+6)+6$

$\therefore g(g(x)) = x+12$

So $g(g(3)) = (3)+12 = 15$

Ex 2: If $f(x) = \sqrt{x-1}$ & $g(x) = x^2$, find $(g \circ f)(x)$ and state its domain.

Soln: $(g \circ f)(x) = g(f(x))$

$= g(\sqrt{x-1})$

$= (\sqrt{x-1})^2$

$\therefore (g \circ f)(x) = x-1$

Domain:

domain of $f(x) = \sqrt{x-1}$ is $x \geq 1$ ($\because x-1 \geq 0$)

domain of $(g \circ f)(x) = x-1$ is $x \in \mathbb{R}$ BUT...

Must consider BOTH the domain of inner fn (ie $f(x)$) and that of composite fn (ie $(g \circ f)(x)$) & combine them:

mean what is common to both

$\therefore$ domain of $(g \circ f)(x)$ is $\{x \mid x \geq 1, x \in \mathbb{R}\}$

Ex 3: If $h(x) = f(g(x))$, determine $f(x)$ & $g(x)$ for

a) $h(x) = (x-2)^2 + (x-2) + 1$ b) $h(x) = \sqrt{x^3+1}$

Soln a) notice $x-2$ repeats – hint!

$\therefore$ it's the inner fn

so let $\boxed{g(x) = x-2}$

$h(x) = (x-2)^2 + (x-2) + 1$

$\therefore f(g(x)) = (g(x))^2 + (g(x)) + 1$

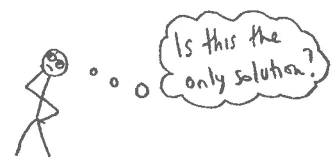
$\therefore \boxed{f(x) = x^2 + x + 1}$

b) let $\boxed{g(x) = x^3+1}$ & work backwards!

$h(x) = \sqrt{x^3+1}$

$f(g(x)) = \sqrt{g(x)}$

$\therefore \boxed{f(x) = \sqrt{x}}$



HWK: pg 507–509 # 2, 3, 5–7, 12, 14a, 17, 20a