Day 14. Review

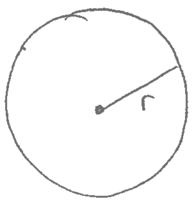
① A Ferris Wheel with a radius of 25.3 m makes two rotations per minute

a) find angular speed (rad/s).

b) How far does a rider travel in 5 min?

② Write an expression that represents all angles coterminal with 30°. Do this in degree & radian measure.

Sol'n ①



$r = 25.3\,m$

speed = 2 rotations/min

recall: speed $= \frac{\text{distance}}{\text{time}}$

a) Angular Speed $= \frac{2 \text{ rotations}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{1 \text{ rotation}} \times \frac{1 \text{ min}}{60\,s}$

$= 0.21$ rad/s

notice how unit analysis makes it easy to determine what to multiply and divide!

b) $C = 2\pi r$

$= 2\pi(25.3)$

$\approx 158.965\,m$

distance $= \frac{158.965\,m}{1 \text{ rotation}} \times \frac{2 \text{ rotations}}{1 \text{ min}} \times \frac{5 \text{ min}}{1}$

$\approx 1590\,m$

② $30° + 360°k$, $k \in \mathbb{I}$ (set of integers ... you can also use $\mathbb{Z}$)

or $30° \pm 360°n$, $n \in \mathbb{N}$ (set of natural numbers)

and

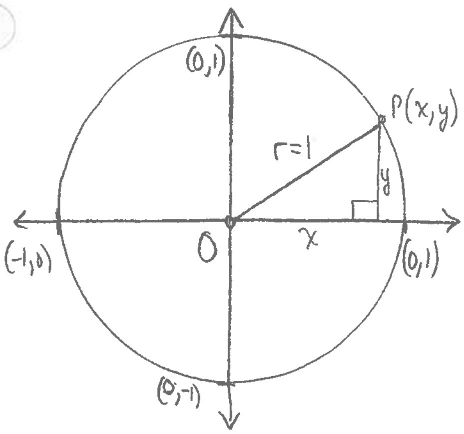
$30° \times \frac{\pi \text{ rad}}{180°} = \frac{30\pi}{180} = \frac{\pi}{6}$

$\therefore \frac{\pi}{6} + 2\pi k$, $k \in \mathbb{Z}$

or $\frac{\pi}{6} \pm 2\pi n$, $n \in \mathbb{N}$

4.2 Unit Circle

Unit stands for ONE. The Unit Circle is a circle with radius 1 unit with centre at the origin.



Notice any point on the unit circle, P(x,y), can be stated using the Pythagorean relationship (see left):

$$x^2 + y^2 = r^2 \quad \text{w/ } r=1$$

$$\boxed{x^2 + y^2 = 1}$$ ← equation for unit circle w/ centre (0,0)

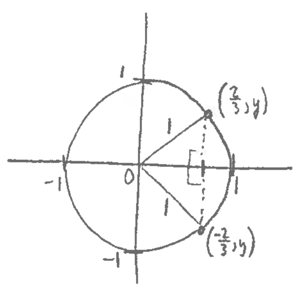
Ex1 Determine all points on the unit circle that satisfy:

a) x - coordinate is $\frac{2}{3}$

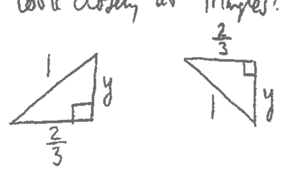
b) y - coordinate is $-\frac{1}{\sqrt{2}}$ & the point is in quadrant III

Draw a diagram in each case.

Sol^n: a)



look closely at triangles:

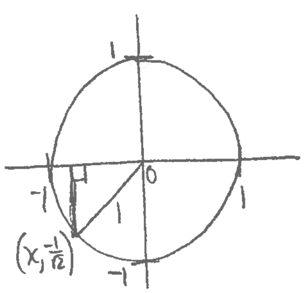


$$x^2 + y^2 = 1$$
$$\left(\frac{2}{3}\right)^2 + y^2 = 1$$
$$\frac{4}{9} + y^2 = 1$$
$$y^2 = \frac{5}{9}$$
$$y = \pm\frac{\sqrt{5}}{3}$$

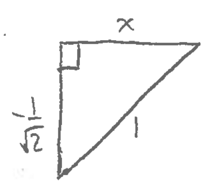
Note: pt in quad I is $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

pt in quad II is $\left(\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

b)



up close:



$$x^2 + y^2 = 1$$
$$x^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = 1$$
$$x^2 + \frac{1}{2} = 1$$
$$x^2 = \frac{1}{2}$$
$$x = -\frac{1}{\sqrt{2}}$$ ← Why only 1 answer? (We only quad III!)

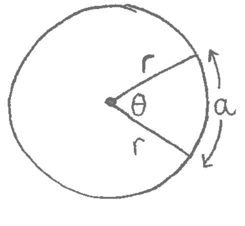
∴ pt in quad III is $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

or $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$

(4.2 continued on next page)

HWK: pg 186-188 # 2a, 3ab, 4a-d, 5a-d, 6, 7, 9-11, 13

Relating Arc length & Angle measure in Radians



recall: $\dfrac{\text{arc length}}{\text{circumference}} = \dfrac{\theta}{2\pi}$

$\Longrightarrow \dfrac{a}{\cancel{2\pi} r} = \dfrac{\theta}{\cancel{2\pi}}$

$\Longrightarrow \dfrac{a}{r} = \theta$

$\Longrightarrow \boxed{a = \theta r}$ ← formula only works when θ measured in radians

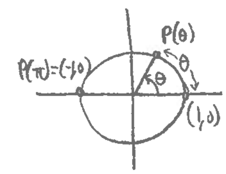
When $r = 1$ $a = \theta(1) = \theta$

i.e $\boxed{a = \theta \text{ for } r = 1}$

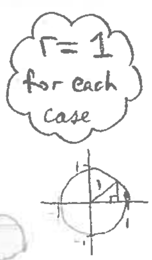
Crazy! I know ☺
(The beauty of radian measure!)

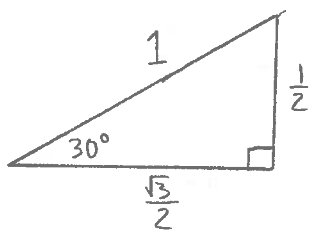
$P(\theta) = (x, y)$ is a fn where P maps $\theta \to (x, y)$

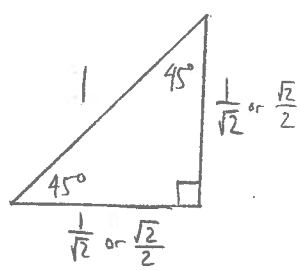
ex $\theta = \pi$, the pt is $(-1, 0)$ ie $P(\pi) = (-1, 0)$



Special Triangles:





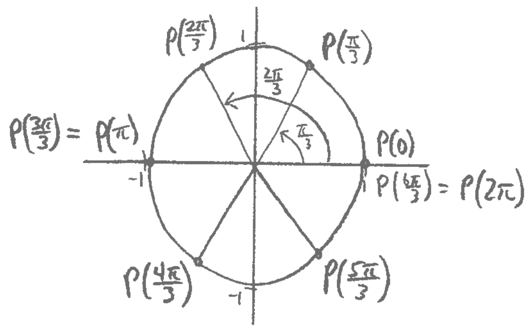


Ex a) On a diagram of the unit circle, show all integral multiples of $\frac{\pi}{3}$ in the interval $0 \le \theta \le 2\pi$

b) What are the coordinates for each $P(\theta)$ in a)?

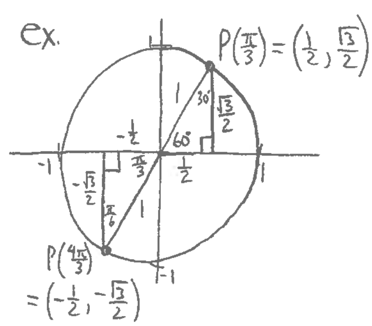
c) Identify any patterns in the pts.

a) Think of it like a pizza or pie:



b) Use the 30°-60°-90° special triangle
$\left(\frac{\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2}\right)$

ex.



$P(0) = (0, 1) = P(2\pi)$ also ☺

$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$P\left(\frac{2\pi}{3}\right) = \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

$P(\pi) = (-1, 0)$

$P\left(\frac{4\pi}{3}\right) = \left(\frac{-1}{2}, -\frac{\sqrt{3}}{2}\right)$

$P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

c) points with angles that are multiples that can't simplify have same coordinates except for signs.
Points where θ reduces to multiple of π fall on an axis