Day 16 Review

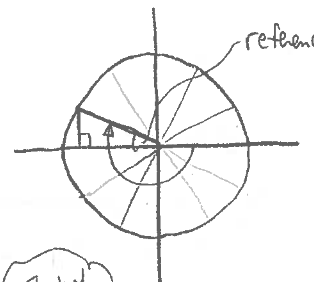
① Determine the exact value of $\sec\left(\frac{-7\pi}{6}\right)$

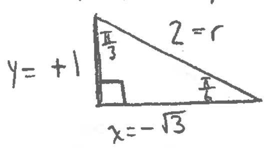
② Determine measures of all angles satisfying $\tan\theta = \sqrt{3}$, $-2\pi \le \theta \le 2\pi$

③ The point $(6,-3)$ lies on the terminal arm of an angle θ in standard position. Find the exact values of the 6 trigonometric ratios for θ.

Sol^n: ① Draw a picture, as always ☺

reference $\angle = \frac{\pi}{6} \Rightarrow$ special triangle!





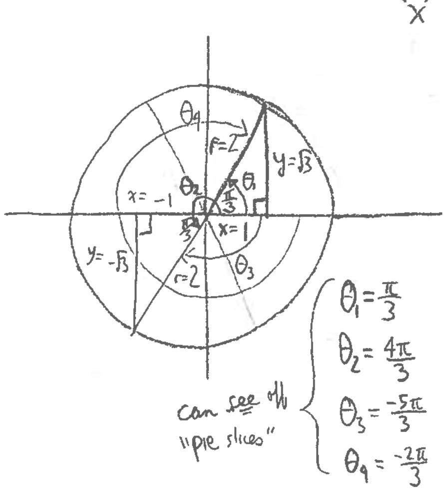
Think of pie slices to visualize $\angle \frac{-7\pi}{6}$

So $\sec\left(\frac{-7\pi}{6}\right) = \frac{r}{x} = \frac{2}{-\sqrt{3}}$

rationalize den. $\frac{2}{-\sqrt{3}}$ if you want: $\frac{2}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$ ($\frac{\sqrt{3}}{\sqrt{3}} = 1$)

$\therefore \sec\left(\frac{-7\pi}{6}\right) = \frac{-2\sqrt{3}}{3}$

② $\tan\theta = \sqrt{3} = \frac{\sqrt{3}}{1}$ or $\frac{-\sqrt{3}}{-1}$ ($\frac{y}{x}$)

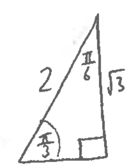


$\theta_1 = \frac{\pi}{3}$
$\theta_2 = \frac{4\pi}{3}$
$\theta_3 = \frac{-5\pi}{3}$
$\theta_4 = \frac{-2\pi}{3}$

can see all "pie slices"

CAST confirms quad I & III
($\because \tan\theta = \sqrt{3} > 0$)

Special △:



reference $\angle = \frac{\pi}{3}$

Algebraically...

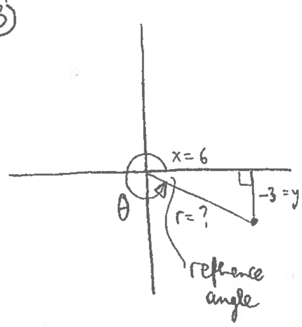
$\theta_1 = \frac{\pi}{3}$ using CAST (quad I & III) $\Rightarrow \theta_2 = \pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$

$\theta_3 = \frac{4\pi}{3} - 2\pi = \frac{4\pi}{3} - \frac{6\pi}{3} = \frac{-2\pi}{3}$ & $\theta_4 = \frac{\pi}{3} - 2\pi = \frac{\pi}{3} - \frac{6\pi}{3} = \frac{-5\pi}{3}$

Co-terminal with θ_2 & θ_1

$\therefore \theta = \frac{-5\pi}{3}, \frac{-2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$ are sol^{ns} on $[-2\pi, 2\pi]$

③



Sides related by Pythagorus:

$x^2 + y^2 = r^2$
$(6)^2 + (-3)^2 = r^2$
$36 + 9 = r^2$
$45 = r^2$
$\therefore \sqrt{45} = r$

rationalizing denominator:

$r = \sqrt{45}$
$= \sqrt{9 \times 5}$
$= \sqrt{9}\sqrt{5}$
$= 3\sqrt{5}$

6 Trig ratios:

$\sin\theta = \frac{y}{r} = \frac{-3}{3\sqrt{5}} = \frac{-1}{\sqrt{5}} \Rightarrow \csc\theta = -\sqrt{5}$

$\cos\theta = \frac{x}{r} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} \Rightarrow \sec\theta = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-3}{6} = -\frac{1}{2} \Rightarrow \cot\theta = -2$

4.4 Intro trig equations

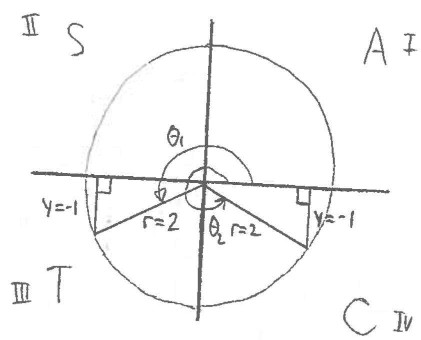
Ex1 Solve the trig equation: $5\sin\theta + 2 = 1 + 3\sin\theta$, $\underbrace{\theta \in [0, 2\pi)}$

this notation means $0 \le \theta < 2\pi$ → $[0, 2\pi)$

Sol^n: Solve like any other algebraic equation ... until the final analysis....

$$5\sin\theta + 2 = 1 + 3\sin\theta$$
$$5\sin\theta + 2 - 3\sin\theta = 1 + 3\sin\theta - 3\sin\theta$$
$$2\sin\theta + 2 = 1$$
$$2\sin\theta + 2 - 2 = 1 - 2$$
$$\frac{2\sin\theta}{2} = \frac{-1}{2}$$
$$\sin\theta = \frac{-1}{2}$$

Analyse with a picture:

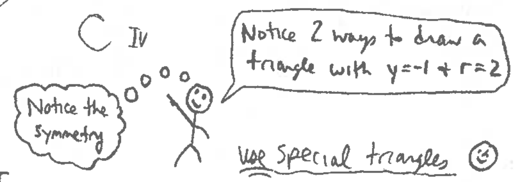


$\sin\theta = \frac{-1}{2} < 0$

$\therefore$ θ in quad III & IV using CAST rule

ratio: $\frac{-1}{2}$ think of: $y = -1$, $r = 2$

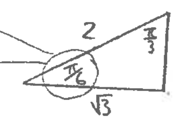
(since $\sin\theta = \frac{y}{r}$)



After analysing, we see that:

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

use special triangles ☺



So $\theta_1 = \pi + \frac{\pi}{6}$
$= \frac{6\pi}{6} + \frac{\pi}{6}$
$= \frac{7\pi}{6}$

and $\theta_2 = 2\pi - \frac{\pi}{6}$
$= \frac{12\pi}{6} - \frac{\pi}{6}$
$= \frac{11\pi}{6}$

Sometimes we must factor but otherwise it's the same kind of analysis:

Ex 2 Solve $\tan^2\theta - 5\tan\theta + 4 = 0$, $0 \le \theta < 2\pi$

$\boxed{\text{note: } \tan^2\theta \text{ means } (\tan\theta)^2}$

Factor: $(\tan\theta - 4)(\tan\theta - 1) = 0$ → Not easy to factor like this? Solve an easier problem...

Let $x = \tan\theta$

Then $\tan^2\theta - 5\tan\theta + 4 = 0$ becomes:

$$x^2 - 5x + 4 = 0$$

factor: $(x-4)(x-1) = 0$

but $x = \tan\theta$, so substitute back:

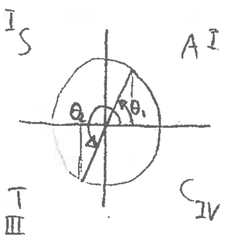
$$(\tan\theta - 4)(\tan\theta - 1) = 0$$

then $\tan\theta - 4 = 0$ -or- $\tan\theta - 1 = 0$

$\tan\theta = 4$ not special ratio

$\tan\theta = 1 = \frac{1}{1}$ or $\frac{-1}{-1} > 0$ $\therefore$ quad I + III

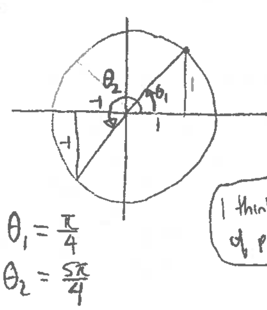
$\therefore$ $\tan^{-1}(\tan\theta) = \tan^{-1}(4)$



notice symmetry!

$\theta_1 \approx 1.3258 > 0$ $\therefore$ quad I + III

$\theta_2 \approx \pi + 1.3258$
≈ 4.4674



$\theta_1 = \frac{\pi}{4}$

$\theta_2 = \frac{5\pi}{4}$

Ex3 Solve for x on $[0, 4\pi)$ if $\sin^2 x - 1 = 0$. What is the general solution (if $x \in \mathbb{R}$)?

Sol^n: $\sin^2 x - 1 = 0$

method I – diff of squares

$(\sin x - 1)(\sin x + 1) = 0$

$\sin x - 1 = 0$ or $\sin x + 1 = 0$

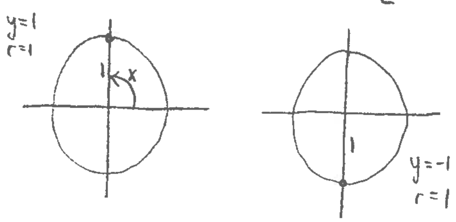
$\sin x = 1 = \frac{1}{1}$ $\quad$ $\sin x = -1 = \frac{-1}{1}$

$\therefore x = \frac{\pi}{2}$ $\quad$ $\therefore x = \frac{3\pi}{2}$

method II – square root

$\sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$



for $x \in [0, 4\pi)$, then we have

$x = \frac{\pi}{2} + 2\pi$ & $x = \frac{3\pi}{2} + 2\pi$ also

$= \frac{\pi}{2} + \frac{4\pi}{2}$ $\quad$ $= \frac{3\pi}{2} + \frac{4\pi}{2}$

$= \frac{5\pi}{2}$ $\quad$ $= \frac{7\pi}{2}$

i.e. $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

General solution involves all coterminal angles of those found.

i.e. $x = \frac{\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$

& $x = \frac{3\pi}{2} + 2k\pi$, $k \in \mathbb{Z}$

(note that these include $\frac{5\pi}{2}$ & $\frac{7\pi}{2}$)

But we can be even <u>more</u> concise:

$$\boxed{x = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}}$$ probably simplest ☺

or as <u>odd multiples</u> of $\frac{\pi}{2}$:

$$x = (2k+1)\left(\frac{\pi}{2}\right), \quad k \in \mathbb{Z}$$

HWR: p211-213 # 5, 7-12, 18, 19