Day 17 Review:

Solve for θ over interval $[0, 2\pi]$. Give exact values where possible. Then find the general solution.

$$\tan^2\theta = 4\tan\theta$$

Soln: $\tan^2\theta = 4\tan\theta$

$\tan^2\theta - 4\tan\theta = 0$

$\tan\theta(\tan\theta - 4) = 0$

recall $\tan\theta = \frac{y}{x}$

$\therefore \tan\theta = 0$ or $\tan\theta - 4 = 0$

$\therefore \theta = \tan^{-1}(0)$
$= 0$

$\tan\theta = 4$

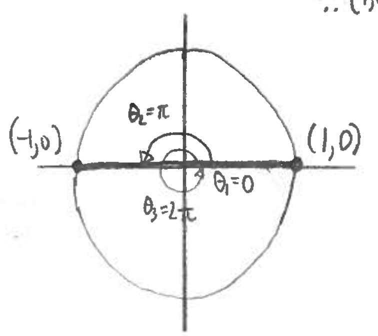
$\therefore \theta = \tan^{-1}(4)$
≈ 1.3258

Analysis: $\tan\theta = \frac{0}{1}$ ← y, ← x or $\frac{0}{-1} = 0$ too

$\therefore (1,0)$ + $(-1,0)$

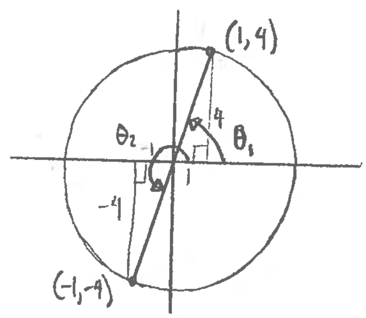
Analysis: $\tan\theta = 4 = \frac{4}{1} = \frac{-4}{-1}$ (y, x)

$\therefore$ pts $(1,4)$ + $(-1,-4)$



get all solns on $[0, 2\pi]$

$\theta_1 = 0$
$\theta_2 = \pi$
$\theta_3 = 2\pi$



$\theta_1 \approx 1.3258$

$\theta_2 \approx \pi + 1.3258$
≈ 4.4674

$\therefore \theta = 0, 1.3258, \pi, 4.4674, 2\pi$ on $[0, 2\pi]$

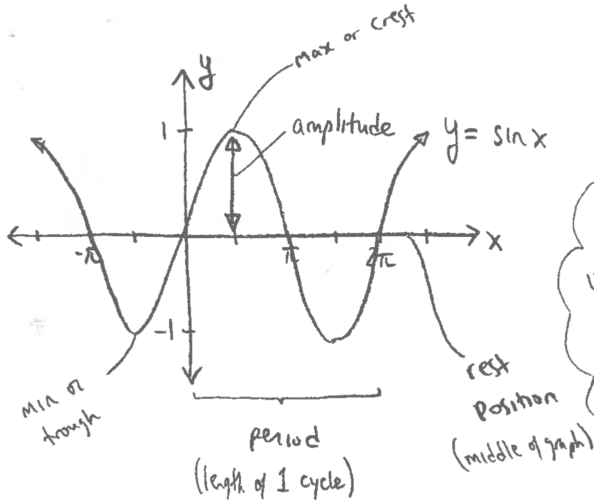
General solution:

Notice (above, left) θ is multiples of π $\Rightarrow$ $\theta = k\pi$, k is any integer

Notice (above, right) θ is 1.3258 plus integer multiples of π $\Rightarrow$ $\theta = 1.3258 + k\pi$, k is any integer

5.1 – Graphing Sine & Cosine Fns

The trig f^{ns} are examples of <u>periodic functions</u> b/c they repeat over a particular interval called the <u>period</u>.



YouTube: Unit circle and sine wave to see where this curve comes from!

ex $y = \sin x$ has period 2π

The <u>amplitude</u> of a periodic f^n is half the difference of its min & max value

ie $\text{Amplitude} = \dfrac{|\max - \min|}{2}$

ex $y = \sin(x)$ has amplitude $\dfrac{|1-(-1)|}{2} = 1$

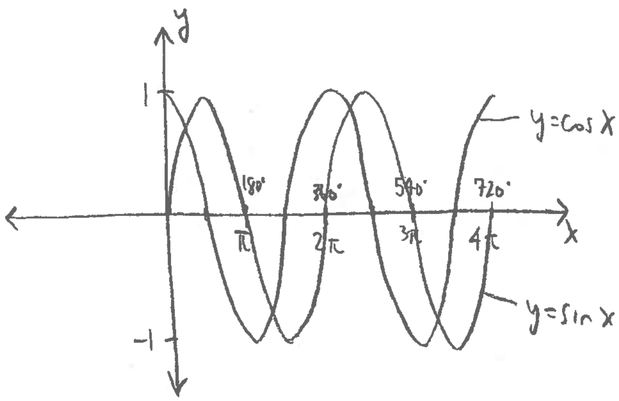
Both the period & amplitude are easily found by examining the graph of the f^n.

Ex 1 Sketch graph of $y = \sin x$ & $y = \cos x$ for $0 \le x \le 4\pi$ (360°)   (both called <u>sinusoidal</u> fns)

a) What is the period & amplitude of each graph?

b) How is the graph of $y = \sin x$ related to the graph of $y = \cos x$?

sol^n



a) period for both is 2π or 360°

amplitude of both is 1

b) $y = \sin x$ is related to $y = \sin x$ by a shift of $\frac{\pi}{2}$ or 90° to the left.

$\therefore \cos x = \sin\left(x + \frac{\pi}{2}\right)$

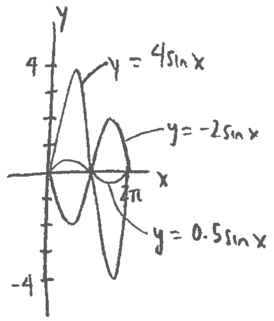
or

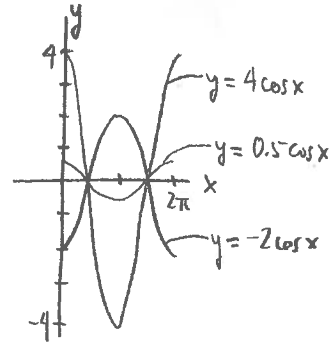
$\cos x = \sin(x + 90°)$

Ex 2 Amplitude of $y = a\sin x$ + $y = a\cos x$. Use graphing calc for a values below. In general, what is the amplitude of these fns?

a) $a = 0.5$ b) $a = 4$ c) $a = -2$

Soln:





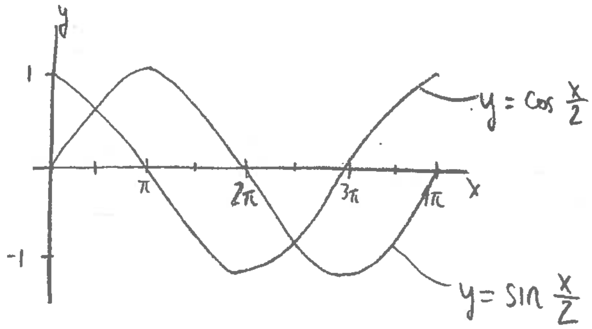
In general, $|a|$ is the amplitude for $y = a\sin x$ + $y = a\cos x$

Ex 3 Period of $y = \sin bx$ + $y = \cos bx$. Use graphing calc for b values below. In general, what is the period of these fns?

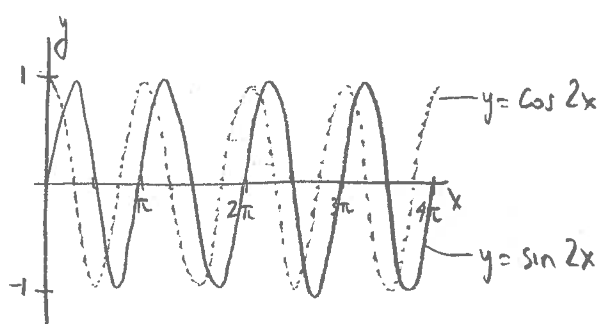
a) $b = 0.5$ b) $b = 2$

hint: try a greater range for x eg $0 \le x \le 4\pi$

Soln:



In general, the period is $\frac{2\pi}{|b|}$ for $y = \sin bx$ + $y = \cos bx$



Ex 4 Predict the amplitude + period for:

a) $y = 2\cos\frac{x}{3}$ b) $y = \frac{-\sin 4x}{3}$

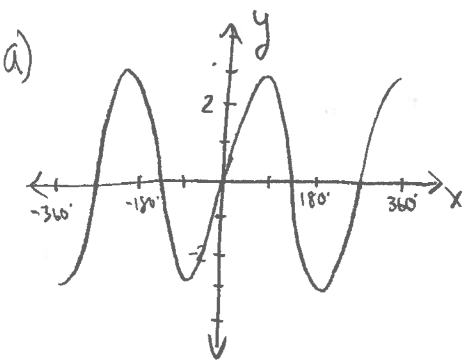
recall: $y = a\sin bx$
$|a|$ is amplitude
$\frac{2\pi}{|b|}$ is period

Soln: a) amplitude is 2
period is $\frac{2\pi}{3}$

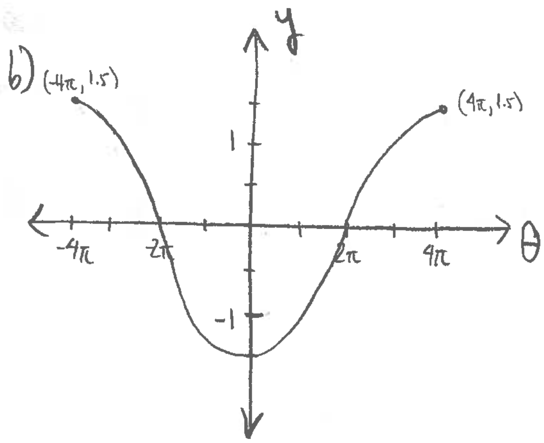
b) amplitude is $\left|\frac{-1}{3}\right|$ or $\frac{1}{3}$
period is $\frac{2\pi}{\frac{1}{3}}$ or 6π

Ex 5 Write an Eq^ from the graph.

a)



b)



Sol^n: amplitude $= \frac{|3-(-3)|}{2} = 3$

period $= \frac{360°}{b} \Rightarrow 270° = \frac{360°}{b}$

$\Rightarrow b = \frac{360°}{270°}$

$= \frac{4}{3}$

We can model this eq^ w/ sine b/c it passes through (0,0)

$\therefore\ y = 3\sin\left(\frac{4x}{3}\right)$

Sol^n: amplitude $= \frac{|1.5-(-1.5)|}{2} = 1.5$

period $= \frac{2\pi}{b} \Rightarrow 8\pi = \frac{2\pi}{b}$

$\Rightarrow b = \frac{2\pi}{8\pi}$

$= \frac{1}{4}$

We can model this eq^ w/ cosine b/c its max/min occurs @ $\theta = 0$

* note that $y = 1.5\cos\frac{\theta}{4}$ is the reflection of this graph through the x-axis

$\therefore$ our graph is $y = -1.5\cos\frac{\theta}{4}$

☐ Hwk: p 233-234 # 6-10 (for questions a-d, you can pick 2 appropriate letters to do for each)

* also... sketch graphs for #9 as practice recommended

5.2 Transformations of Sinusoidal Fns

Transformations:

$$y = a \sin b(x-c) + d$$

vertical stretch — a

horizontal stretch — b

horizontal translation — c

vertical translation — d

Sinusoidal Fns:

$|a|$ is amplitude

$\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$ is period

phase shift:
$c>0 \Rightarrow c$ units right
$c<0 \Rightarrow c$ units left

vertical displacement:
$d>0 \Rightarrow d$ units up
$d<0 \Rightarrow d$ units down

Ex1 Consider $y = 3 \sin 2\left(t - \frac{\pi}{4}\right) + 2$

a) determine the domain, range, amplitude, period, y-intercept, phase shift, and vertical displacement with respect to $y = \sin x$.

b) Determine the x-intercepts on $[0, 2\pi]$ using a graphing calculator.

c) sketch this function over one or more complete cycles.

Sol^n: a) unrestricted domain of all sinusoidal Fns is the Real numbers

$\therefore$ domain: $t \in \mathbb{R}$

amplitude is $|a| = |3| = 3$

vertical displacement is $d = 2$
i.e., 2 units up

phase shift from $c = \frac{\pi}{4}$
i.e., $\frac{\pi}{4}$ units right

Use rest position (d value) and $\pm$ amplitude ($|a|$ value) to get

range: min: $2-3=-1$
max: $2+3=5$

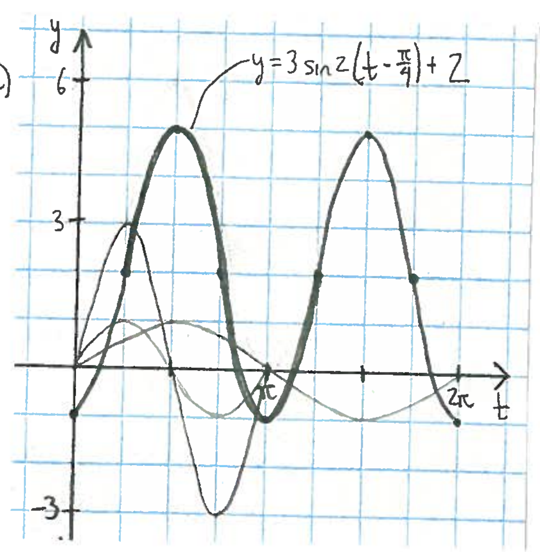
$\therefore$ range is $-1 \le y \le 5$

b) use $Y1 = 3\sin(2(X-(\pi/4)))+2$ in graphing calculator

Then $\boxed{\text{Zero}}$ Function to get roots (x-intercepts) on $[0, 2\pi]$

$x \approx 0.4204,\ 2.7210,\ 3.5621,\ 5.8627$.

c)



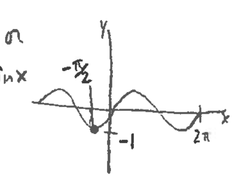
Use info from a) and your vast knowledge of transformations to graph!

What if I moved the axes to suit my graph? Or even changed the axes?

y-int: set $t=0 \Rightarrow y = 3\sin 2\left(0-\frac{\pi}{4}\right)+2$
$= 3\sin\left(-\frac{\pi}{2}\right)+2$
$= 3(-1)+2$
$= -1$

$\therefore$ y-int is -1

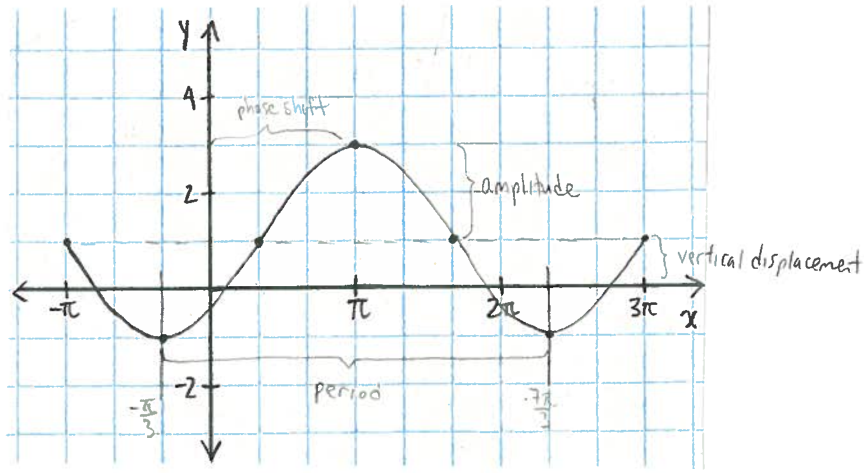
use calculator or graph of $y=\sin x$



period $= \frac{2\pi}{|b|}$
$= \frac{2\pi}{2}$
$= \pi$

$\therefore$ period is π

Ex 2 Determine an equation of the form $y = a\cos b(x-c) + d$ from the graph below.



Notice (as labelled in graph), that we can determine: amplitude, period, phase shift, and vertical displacement right from the graph. These correspond to a, b, c, and d values, respectively.

amplitude is 2. We can use formula if necessary: $\frac{|\max - \min|}{2} = \frac{|3-(-1)|}{2} = \frac{|4|}{2} = 2$

$\Longrightarrow \boxed{a = 2}$

period is $\frac{8\pi}{3}$. We can see this from graph: $\frac{\pi}{3} + 2\pi + \frac{\pi}{3} = \frac{1\pi}{3} + \frac{6\pi}{3} + \frac{1\pi}{3} = \frac{8\pi}{3}$ (visually)

or: $\frac{7\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{8\pi}{3}$

Use $\text{period} = \frac{2\pi}{|b|} \Longrightarrow \frac{\cancel{8}^{4}\pi}{3} = \frac{\cancel{2}^{1}\pi}{b}$

$\frac{4}{3} = \frac{1}{b}$ (divide both sides by 2π)

$\therefore \boxed{\frac{3}{4} = b}$ (reciprocal of both sides)

phase shift is π units to the right relative to graph of $y = \cos x$

$\Longrightarrow \boxed{c = \pi}$

The appropriate phase shift will take care of any reflections — why?

vertical displacement is 1 unit up relative to graph of $y = \cos x$

$\Longrightarrow \boxed{d = 1}$

Put it all together: $y = 2\cos\frac{3}{4}(x-\pi) + 1$

How could I have done this with a sine equation?

☐ HWR: p 233-234 # 3, 5, 6a, 7a, 10, 11a, 13–16