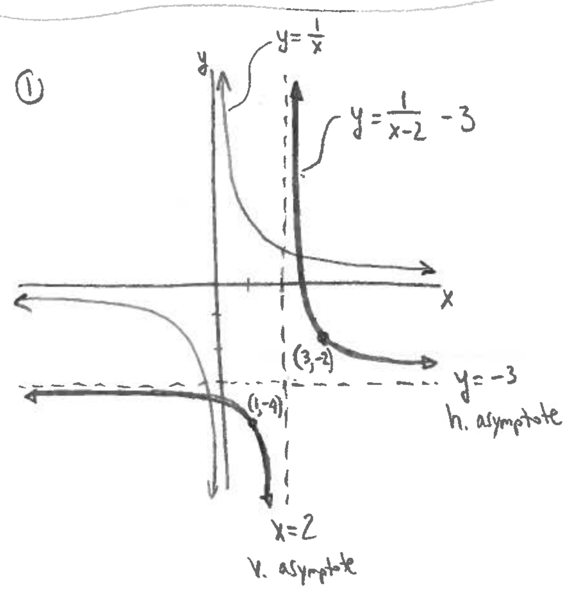
Day 2 Review

① Use transformations to sketch the graph of $y = \frac{1}{x-2} - 3$, starting from $y = \frac{1}{x}$

② If $(7, 12)$ is a point on the graph of $y = f(x)$, find a point on the graph of $y = f(x+6) + 8$.

Sol^n: ①



recall $y = f(\underbrace{x-h}_{\text{opposite}}) + \underbrace{k}_{\text{deals w/ y values only}}$

Opposite
deals w/ x values only
⊕ left
⊖ right

deals w/ y values only
⊕ up
⊖ down

So $y = \frac{1}{x\underbrace{-2}_{\text{right 2}}} \underbrace{-3}_{\text{down 3}}$ from $y = \frac{1}{x}$

② $y = f(x\underbrace{+6}_{\text{left 6}}) \underbrace{+8}_{\text{up 8}}$

left 6
i.e. subtract 6 from x value

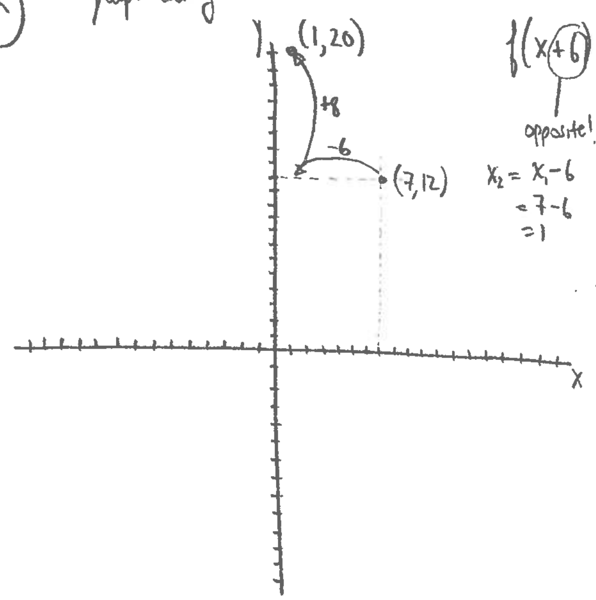
up 8
i.e. add 8 to y value

So $(7,12) \to x = 7$, $y = 12$

becomes: $x = 7 - 6 = 1$, $y = 12 + 8 = 20$

to get $(1, 20)$

OR graphically:



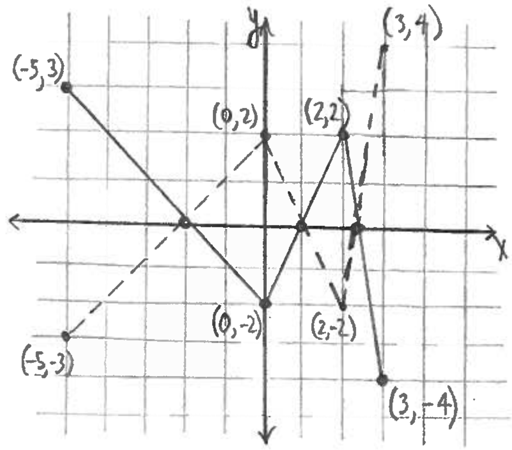
$f(x\underbrace{+6}_{\text{opposite!}}) \underbrace{+8}$

opposite!
$x_2 = x_1 - 6$
$= 7 - 6$
$= 1$

$y_2 = y_1 + 8$
$= 12 + 8$
$= 20$

1.2 Reflections & Stretches

Ex1 Given the graph of $y = f(x)$, graph $y = -f(x)$ on the same axes. Describe how they are related.



Soln Can use table of values:

$y = f(x)$

x	y
-5	3
0	-2
2	2
3	-4

$y = -f(x)$

x	y
-5	-3
0	2
2	-2
3	4

change sign!

OR notice that $y = f(x)$ tells us "$f(x)$" is the y-value so that $y = -f(x)$ tells us the y-values are negated (i.e. change sign)

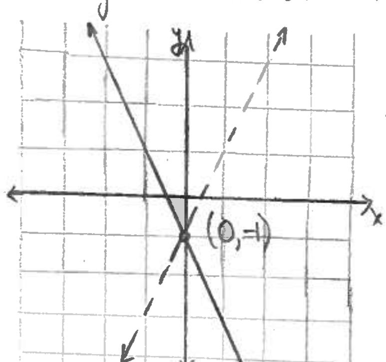
Graphically we can see that we replot the points with x-values constant and opposite y-values.

So $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis. (The x-axis is the mirror line.)

* The points along the x-axis (the line of reflection) are INVARIANT (they stay the same).
Ex $(-2,0)$; $(1,0)$; $(2\frac{1}{3},0)$ above

Ex 2 Let $f(x) = -2x-1$. Graph $y = f(x)$ and $y = f(-x)$ on the same axes. Describe how they are related. Write an equation for $y = f(-x)$ as well. Label any invariant points.

Soln:



$$y = f(-x)$$
$$= -2(-x)-1$$
$$= 2x-1$$

use a graphing calculator or plot yourself

$y=2x-1$

x	y
-1	-3
0	-1
1	1
2	3

$y=-2x-1$

x	y
1	-3
0	-1
-1	1
-2	3

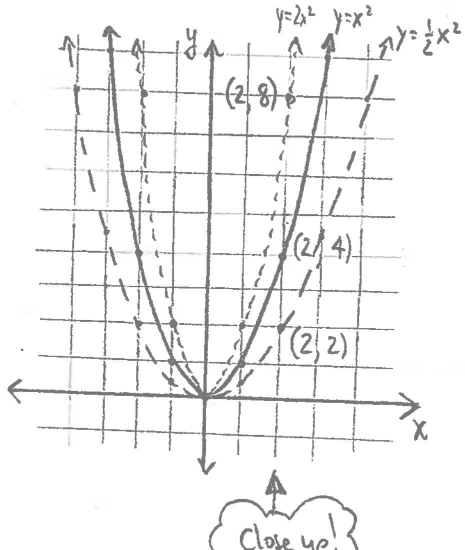
note sign change in x values

Graphically, we see:
- y-values are constant and
- x values are the opposite sign
- invariant points lie on y-axis (mirror line)

So $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

STRETCHES

Ex 1 Graph the functions $y = x^2$, $y = 2x^2$, and $y = \frac{1}{2}x^2$ on the same axes. How are the last two related to the first?



notice: • y-coordinate $\frac{1}{2}$ for $y=\frac{1}{2}x^2$ than $y=x^2$
• y-coordinate double for $y=2x^2$ than $y=x^2$

$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

$y = 2x^2$

x	y
-3	2(9) = 18
-2	2(4) = 8
-1	2(1) = 2
0	2(0) = 0
1	2(1) = 2
2	2(4) = 8
3	2(9) = 18

y-values (x2)
x-values same

$y = \frac{1}{2}x^2$

x	y
-3	$\frac{1}{2}(9) = 4.5$
-2	$\frac{1}{2}(4) = 2$
-1	$\frac{1}{2}(1) = 0.5$
0	$\frac{1}{2}(0) = 0$
1	$\frac{1}{2}(1) = 0.5$
2	$\frac{1}{2}(4) = 2$
3	$\frac{1}{2}(9) = 4.5$

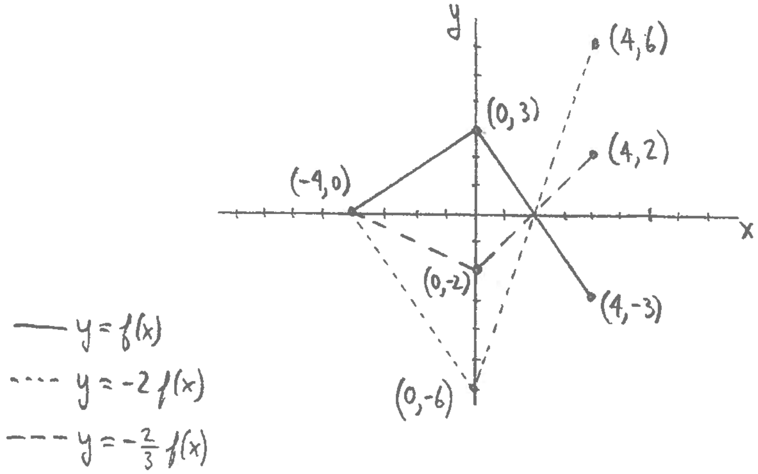
y-values ($\times\frac{1}{2}$)
x-values same

Graphically, $y = 2x^2$ is expanded vertically by factor 2 from the graph of $y = x^2$
$y = \frac{1}{2}x^2$ is compressed vertically by factor $\frac{1}{2}$ from the graph of $y = x^2$

In general, the graph of $y = a\,f(x)$, a is constant, is related to the graph of $y = f(x)$ by:

- a vertical expansion of factor $|a|$, when $|a| > 1$
- a vertical compression of factor $|a|$, when $0 < |a| < 1$ ← means $-1 < a < 0$ or $0 < a < 1$

Try: Ex 2. Sketch the graph of $y=f(x)$ as shown. On the same set of axes, sketch the graphs of $y=-2f(x)$ and $y=-\frac{2}{3}f(x)$



$y=f(x)$

x	y
-4	0
0	3
4	-3

$y=-2f(x)$

x	y
-4	$-2(0)=0$
0	$-2(3)=-6$
4	$-2(-3)=6$

$y=-\frac{2}{3}f(x)$

x	y
-4	$-\frac{2}{3}(0)=0$
0	$-\frac{2}{3}(3)=-2$
4	$-\frac{2}{3}(-3)=2$

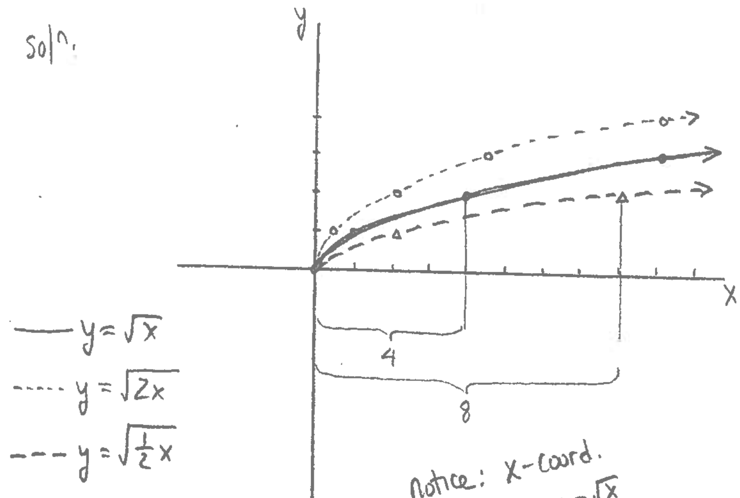
note: x constant

y value multiplied by a value ☺

Use a table of values or perform the transformation on the graph (eventually you won't need tables of values).

Ex 3 Graph the functions $y=\sqrt{x}$, $y=\sqrt{2x}$, $y=\sqrt{\frac{1}{2}x}$ on the same axes.
How are the last two related to the first?

Sol^n.



notice: x-coord. in graph of $y=\sqrt{x}$ are doubled for graph of $y=\sqrt{\frac{1}{2}x}$, y-coord. is constant.

$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3

$y=\sqrt{2x}$

x	y
$0=\sqrt{2(0)}\to 0$	0
$1=\sqrt{2(\frac{1}{2})}\to \frac{1}{2}$	1
$2=\sqrt{2(2)}\to 2$	2
$3=\sqrt{2(\frac{9}{2})}\to \frac{9}{2}$	3

x-values halved
y constant

$y=\sqrt{\frac{1}{2}x}$

x	y
$0=\sqrt{\frac{(0)}{2}}\to 0$	0
$1=\sqrt{\frac{(2)}{2}}\to 2$	1
$2=\sqrt{\frac{(8)}{2}}\to 8$	2
$3=\sqrt{\frac{(18)}{2}}\to 18$	3

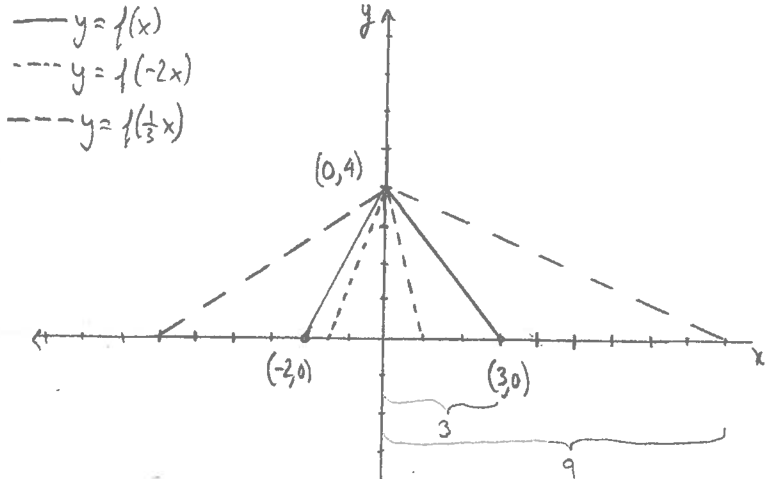
x-values doubled
y constant

So we see graph of $y=\sqrt{2x}$ is compressed horizontally by $\frac{1}{2}$ from graph of $y=\sqrt{x}$ and graph of $y=\sqrt{\frac{1}{2}x}$ is expanded horizontally by factor 2 from graph of $y=\sqrt{x}$

In general, the graph of $y = f(kx)$, k a constant, is related to the graph of $y = f(x)$ by

- a <u>horizontal</u> <u>compression</u> of factor $\left|\frac{1}{k}\right|$, when $|k| > 1$
- a <u>horizontal</u> <u>expansion</u> of factor $\left|\frac{1}{k}\right|$, when $0 < |k| < 1$

Ex 4 The grid below shows the graph of $y = f(x)$. Sketch the graphs of $y = f(-2x)$ and $y = f\left(\frac{1}{3}x\right)$ on the same axes.



$k = 3 \Rightarrow$ x-values multiplied by 3

$y = f(x)$

x	y
-2	0
0	4
3	0

$y = f(-2x)$

x	y
$-\frac{1}{2}(-2) = 1$	0
$-\frac{1}{2}(0) = 0$	4
$-\frac{1}{2}(3) = -\frac{3}{2}$	0

$k = -2$

$\therefore$ multiply x-values by $\frac{1}{k}$ or $\frac{1}{-2}$

y constant

$y = f\left(\frac{1}{3}x\right)$

x	y
$3(-2) = -6$	0
$3(0) = 0$	4
$3(3) = 9$	0

$k = \frac{1}{3}$

$\therefore$ multiply x-values by $\frac{1}{k}$ or $\frac{1}{\frac{1}{3}} = 3$

y constant

* If doing this graphically, you can do the stretch 1st & then the reflection (if it's too hard in 1 step)

Hwk: pg 28-31 # 1-4, 7, 9, 10, 12, 14-16