Day 20

Review:

① Prove one or both of these identities:

a) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$ b) $\dfrac{\cos^2\theta}{1+2\sin\theta-3\sin^2\theta} = \dfrac{1+\sin\theta}{1+3\sin\theta}$

② Simplify: $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$. Confirm your solution graphically. State any restrictions.

③ Solve $\cos^2 x = \cot x \sin x$ algebraically. Verify your solution graphically.

Sol^n: ① a)

LS	RS
$= (\tan x - 1)^2$	$= \sec^2 x - 2\tan x$
$= (\tan x - 1)(\tan x - 1)$	$= (1+\tan^2 x) - 2\tan x$
$= \tan^2 x - 2\tan x + 1$	$= \tan^2 x - 2\tan x + 1$

$\because LS = RS \therefore (\tan x - 1)^2 = \sec^2 x - 2\tan x$ for all permissable values of x.

b)

LS	RS
$= \dfrac{\cos^2\theta}{1+2\sin\theta-3\sin^2\theta}$	$= \dfrac{1+\sin\theta}{1+3\sin\theta}$
$= \dfrac{\cos^2\theta}{(1+3\sin\theta)(1-\sin\theta)}$	
$= \dfrac{1-\sin^2\theta}{(1+3\sin\theta)(1-\sin\theta)}$	
$= \dfrac{(1+\sin\theta)\cancel{(1-\sin\theta)}}{(1+3\sin\theta)\cancel{(1-\sin\theta)}}$	
$= \dfrac{1+\sin\theta}{1+3\sin\theta}$	

$\because LS = RS \therefore \dfrac{\cos^2\theta}{1+2\sin\theta-3\sin^2\theta} = \dfrac{1+\sin\theta}{1+3\sin\theta}$ for all permissable values of x.

② $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$

$= \dfrac{1-\cos x}{(1+\cos x)(1-\cos x)} + \dfrac{1+\cos x}{(1-\cos x)(1+\cos x)}$

$= \dfrac{1-\cancel{\cos x}+1+\cancel{\cos x}}{(1+\cos x)(1-\cos x)}$

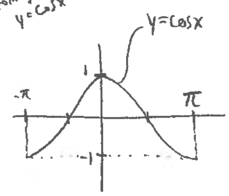
$= \dfrac{2}{1-\cos^2 x}$

$= \dfrac{2}{\sin^2 x}$

$= 2\left(\dfrac{1}{\sin x}\right)^2$

$= 2\csc^2 x$

from graphing $y=\cos x$



restrictions:

$1+\cos x \neq 0 \Rightarrow \cos x \neq -1$

$\Rightarrow x \neq \pi + 2k\pi, k\in\mathbb{Z}$

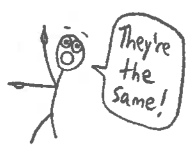
$1-\cos x \neq 0 \Rightarrow \cos x \neq 1$

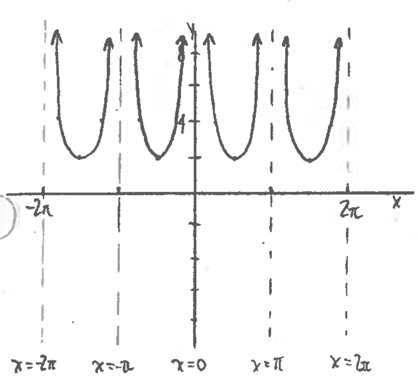
$\Rightarrow x \neq 2k\pi, k\in\mathbb{Z}$

Together: $x \neq k\pi, k\in\mathbb{Z}$

off graph

$\Rightarrow x \neq k\pi, k\in\mathbb{Z}$





← Both graphs are identical, suggesting $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x} = 2\csc^2 x$

Restrictions: $x \neq k\pi, k\in\mathbb{Z}$

③ $\cos^2 x = \cot x \sin x$

$\cos^2 x = \left(\dfrac{\cos x}{\cancel{\sin x}}\right)\dfrac{\cancel{\sin x}}{1}$

$\cos^2 x = \cos x$

$\cos^2 x - \cos x = 0$

$\cos x(\cos x - 1) = 0$

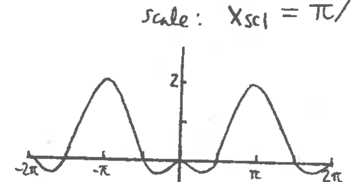
$\therefore \cos x = 0$ or $\cos x - 1 = 0$

$\therefore x = \frac{\pi}{2} + 2k\pi, k\in\mathbb{Z}$ $\cos x = 1$

$\therefore x = 2k\pi, k\in\mathbb{Z}$

check: $Y1 = (\cos(x))^2 - \cos(x)$

use $\boxed{\text{ZERO}}$ function or choose appropriate scale: $X_{scl} = \pi/2$



roots: $\{2k\pi ; \frac{\pi}{2} + 2k\pi, k\in\mathbb{Z}\}$

Sum, Difference, and Double-Angle Identities (6.2 + applications in 6.3 + 6.4)

You can do the Investigation on pgs 299 + 300 of your text to derive the sum identities for $\sin(\alpha+\beta)$ + $\cos(\alpha+\beta)$ ... or checkout YouTube videos ☺ Here they are...

Sum identities:

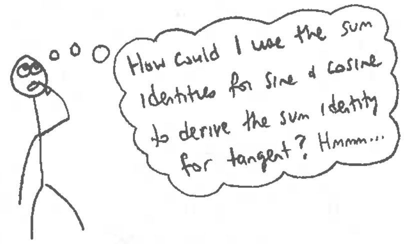
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$$

you can use $(-B)$ in place of B (+ simplify) to derive the difference identities ☺

TRY IT!

Difference Identities:

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$$



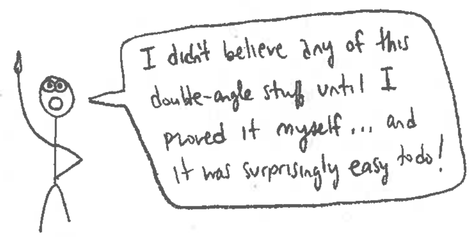
You'll often see them written together as:

$$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A\pm B) = \frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

How are $\pm$ and $\mp$ related? Compare the sum and difference identities above to find out.

If we let $A=B$ in the sum identities, we get a special case we call the double-angle identities:

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$



Ex1 Write each expression as a single trig function.

a) $\sin 48°\cos 17° - \cos 48°\sin 17°$

notice the identity for $\sin(A-B)$ fits here, where $A=48°$ + $B=17°$

$\therefore \sin 48°\cos 17° - \cos 48°\sin 17°$

$= \sin(48°-17°)$

$= \sin 31°$

b) $\cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

Notice the identity for $\cos 2A$ fits here, where $A=\frac{\pi}{3}$

$\therefore \cos^2\frac{\pi}{3} - \sin^2\frac{\pi}{3}$

$= \cos 2\left(\frac{\pi}{3}\right)$

$= \cos\frac{2\pi}{3}$

Ex 2 Write an identity for $\cos 2A$ that contains only:

a) the cosine ratio

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= \cos^2 A - (1-\cos^2 A)\\ &= \cos^2 A - 1 + \cos^2 A\\ &= 2\cos^2 A - 1\end{aligned}$$

b) the sine ratio

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= (1-\sin^2 A) - \sin^2 A\\ &= 1 - 2\sin^2 A\end{aligned}$$

Aside:

$\sin^2 A + \cos^2 A = 1$

So... $\sin^2 A = 1 - \cos^2 A$

and... $\cos^2 A = 1 - \sin^2 A$

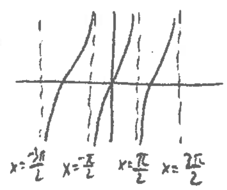
So two other double-angle identities are: $\cos 2A = 2\cos^2 A - 1$ + $\cos 2A = 1 - 2\sin^2 A$

Ex 3 Simplify $\dfrac{1-\cos 2x}{\sin 2x}$ and state any restrictions.

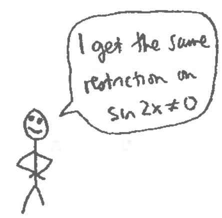
$$\begin{aligned}\frac{1-\cos 2x}{\sin 2x} &= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}\\ &= \frac{1-1+2\sin^2 x}{2\sin x\cos x}\\ &= \frac{\cancel{2\sin x}\cdot\sin x}{\cancel{2\sin x}\cdot\cos x}\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

* Easiest to do restrictions on simplified trig ratio:

$y = \tan x$:



$\therefore\ x \neq \frac{k\pi}{2},\ k \in \mathbb{Z}$



Ex 4 Determine the exact value of $\sin\frac{\pi}{12}$.

Soln:
$$\begin{aligned}\sin\frac{\pi}{12} &= \sin\left(\frac{3\pi}{12}-\frac{2\pi}{12}\right)\\ &= \sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)\end{aligned}$$

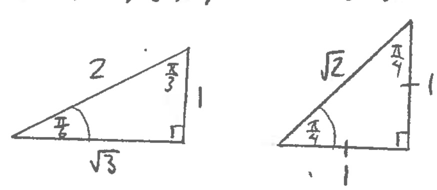
← note I want special angles in order to use the special triangles to get an exact value!

$$\begin{aligned}&= \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}\\ &= \left(\frac{1}{\sqrt2}\right)\left(\frac{\sqrt3}{2}\right) - \left(\frac{1}{\sqrt2}\right)\left(\frac{1}{2}\right)\\ &= \frac{\sqrt3}{2\sqrt2} - \frac{1}{2\sqrt2}\\ &= \frac{\sqrt3 - 1}{2\sqrt2} \times \frac{2\sqrt2}{2\sqrt2}\end{aligned}$$

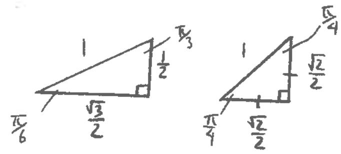
if you'd like ☺

$$\begin{aligned}&= \frac{2\sqrt6 - \cancel{2}\sqrt2}{4\cdot\cancel{2}}\\ &= \frac{\sqrt6-\sqrt2}{4}\end{aligned}$$

$\because (\sqrt2)^2 = 2$



all this work would've been spared by using the unit ratios instead:



Ex 5 Prove the identity: $\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1} = \dfrac{\sin^2 x \cos x + \cos^3 x}{2\sin x + 1}$

LS	RS
$= \dfrac{2\sin x\cos x - \cos x}{4\sin^2 x - 1}$	$= \dfrac{\cos x\,\cancelto{1}{(\sin^2 x + \cos^2 x)}}{2\sin x + 1}$
$= \dfrac{\cos x\,\cancel{(2\sin x - 1)}}{(2\sin x + 1)\cancel{(2\sin x - 1)}}$	$= \dfrac{\cos x}{2\sin x + 1}$
$= \dfrac{\cos x}{2\sin x + 1}$	

$\because$ LS = RS $\therefore$ statement is true for all permissable values of x.

Ex 6 Solve $\sin 2x = \sqrt{2}\cos x$ on $[0, 2\pi)$

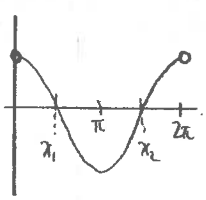
Sol^n $\sin 2x = \sqrt{2}\cos x$

$2\sin x\cos x = \sqrt{2}\cos x$

$2\sin x\cos x - \sqrt{2}\cos x = 0$

$\cos x(2\sin x - \sqrt{2}) = 0$

$\therefore \cos x = 0$ or $2\sin x - \sqrt{2} = 0$

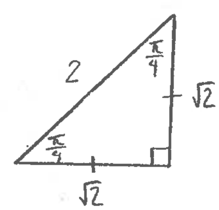


$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$

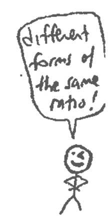
$2\sin x = \sqrt{2}$

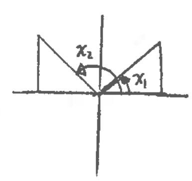
$\sin x = \dfrac{\sqrt{2}}{2}$ $\begin{matrix} = y \\ = r \end{matrix}$

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}$



$\sqrt{2} : \sqrt{2} : 2$
$= \frac{\sqrt{2}}{2} : \frac{\sqrt{2}}{2} : 1$
$= 1 : 1 : \sqrt{2}$





$x_1 = \frac{\pi}{4}$
$x_2 = \frac{3\pi}{4}$

All together... Solution is $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}\right\}$

HWK: 6.2 p 306 # 2a,c; 8a, 18 (challenge: 22)
6.3 p 314 # 10a,c; 11a, 15
6.4 p 321 # 9, 15

Test Prep:

☐ do all of your assignment

☐ p 328-329 # 1-20

☐ Trig Review Pkg on website — with full sol^ns ☺ (under "exam prep")

LOTS! Do one or the other. (you'll get time next class)