Day 21

Review

① Determine the number of solutions for $(a\cos x + b)(c\sin x + d) = 0$, for $x \in [0, 2\pi)$, if $a = b \neq 0$ and $1 < d < c$.

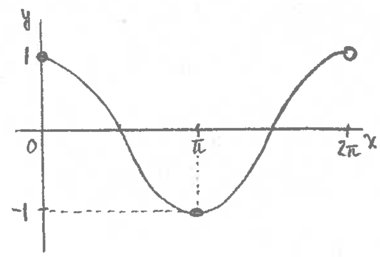
② If $\tan x = \frac{3}{4}$ and $\cos x = \frac{-4}{5}$, find the exact value of $\sin 2x$.

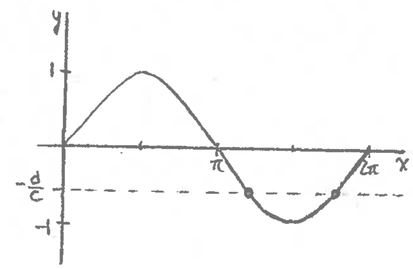
Sol'n: ① If $(a\cos x + b)(c\sin x + d) = 0$,

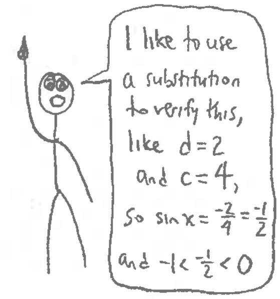
then: $a\cos x + b = 0$ or $c\sin x + d = 0$

$\cos x = \frac{-b}{a}$ $\qquad$ $\sin x = \frac{-d}{c}$

$a = b \neq 0 \Rightarrow \cos x = -1$ $\qquad$ $1 < d < c \Rightarrow -1 < \sin x < 0$







There is only 1 solution on $[0, 2\pi)$
(the solution is π)

There are 2 solutions on $[0, 2\pi)$
(we can see this, even though we don't know the values)

Therefore, there are 3 solutions in total.

② method I:

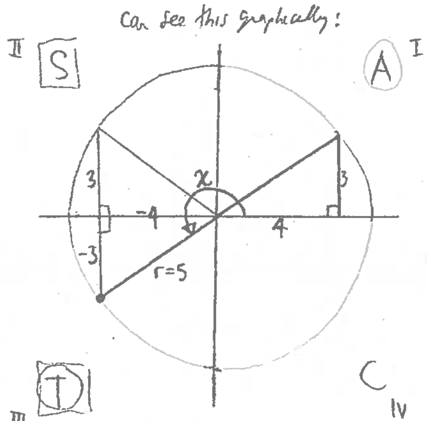
$\tan x = \frac{3}{4} > 0 \Rightarrow x$ is in quad I or III

$\cos x = \frac{-4}{5} < 0 \Rightarrow x$ is in quad II or III

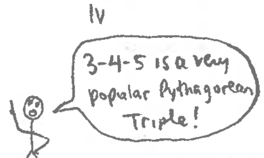
must be in III!

$\therefore x$ is in quad III

Can see this graphically:



$r^2 = (-3)^2 + (-4)^2$
$= 9 + 16$
$= 25$
$\therefore r = \sqrt{25} = 5$



Need to find:

$\sin 2x = 2\sin x\cos x$

$= 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$ from Quad III

$= \frac{24}{25}$

method II:

$\tan x = \frac{\sin x}{\cos x} \Longrightarrow \sin x = \tan x \cos x$

$\therefore \sin 2x = 2\sin x \cos x$

$= 2(\tan x \cos x)\cos x$

$= 2\tan x \cos^2 x$

$= 2\left(\frac{3}{4}\right)\left(\frac{-4}{5}\right)^2$

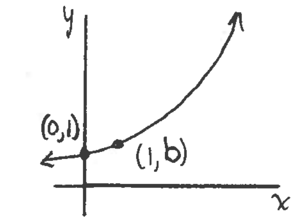
$= \left(\frac{3}{2}\right)\left(\frac{16}{25}\right)$

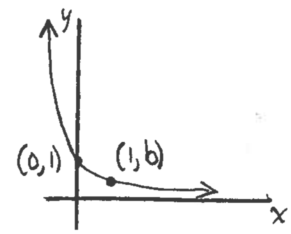
$= \frac{24}{25}$

7.1 Characteristics of Exponential Functions

Exponential FNs are of the form $y = b^x$, where b is the "base" and x is the "exponent".

$y = b^x$ is increasing for $b > 1$ and decreasing for $0 < b < 1$

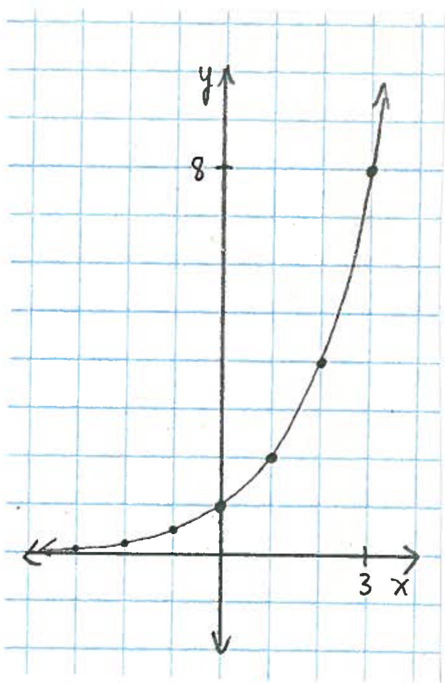


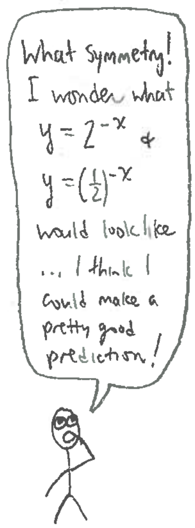


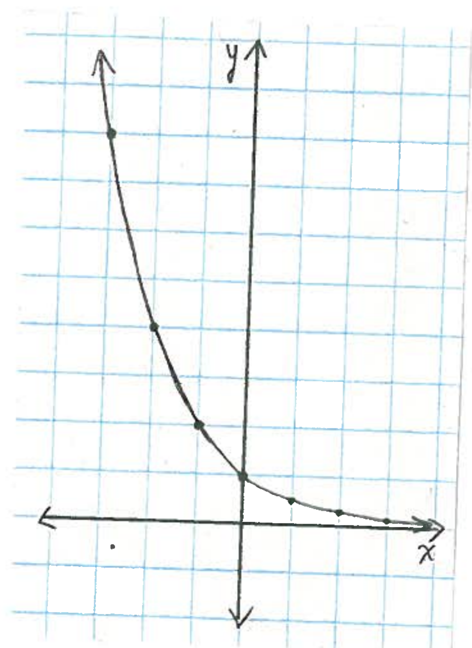
$(b>1)$ increasing (exponential growth) $\qquad$ $(0<b<1)$ decreasing (exponential decay)

Ex 1 Graph the following exponential functions without technology. Discuss the characteristics of the graphs (e.g., domain + range, x + y-intercepts, increasing vs. decreasing FN, asymptotes)

a) $y = 2^x$ $\qquad$ b) $y = \left(\frac{1}{2}\right)^x$







a)

Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

no x-intercept ; y-intercept is 1

$y = 2^x$ is increasing (i.e., as x increases)

horizontal asymptote: $y = 0$

b)

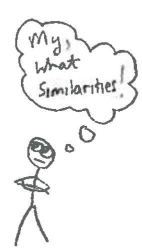
Domain: $\{x \mid x \in \mathbb{R}\}$

Range: $\{y \mid y > 0, y \in \mathbb{R}\}$

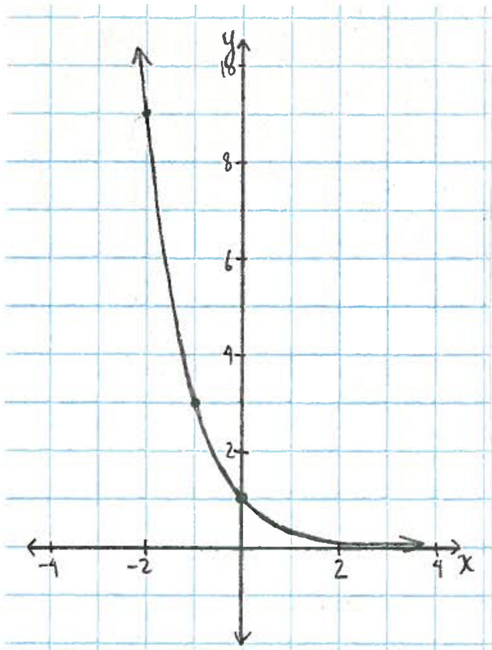
no x-intercept ; y-intercept is 1

$y = (\frac{1}{2})^x$ is decreasing (i.e., as x increases)

horizontal asymptote: $y = 0$



Ex 2. Write an exponential function to describe the graph below.



Soln: use key points in a table of value

ex: $(-2, 9)$, $(-1, 3)$, $(0, 1)$

x	y
-2	9
-1	3
0	1
1	$\frac{1}{3}$

$\times \frac{1}{3}$ $\times \frac{1}{3}$ $\times \frac{1}{3}$

look for a pattern

extrapolate: check on graph!

$\therefore\ y = \left(\frac{1}{3}\right)^x$ fits the graph

-OR-

For exponential FNS of form $y = b^x$, we know $(1, b)$ & $(-1, \frac{1}{b})$ are points. For $0 < b < 1$, $(-1, \frac{1}{b})$ is more useful: $(-1, 3) \Rightarrow b = \frac{1}{3}$ $\therefore\ y = (\frac{1}{3})^x$

Ex 3 A colony of bacteria triples every 30h. The initial count is 100.

a) Write an exponential function to model this scenario.

b) About how many bacteria will there be in 4 days?

c) About how many bacteria were there 12 hours ago?

d) How long will it take for the population to reach 1000?

Soln: a) let $B(t)$ represent the number of bacteria after t hours.

Then $B(t) = 100(3)^{\frac{t}{30}}$

Think: It takes 30h to triple the population, so when $t = 30$, you expect $B(30) = 300$

100 × 3

$\therefore\ 3^{t/30}$ satisfies tripling every 30h:

$B(30) = 100 \cdot 3^{(30)/30} = 100 \cdot 3^1 = 300$

× 3

$\therefore\ B(t) = 100(3)^{t/30}$ fits the scenario

analysis: Initial count means when $t = 0$

Want $B(0) = 100$

We can see that $t = 0$ makes $3^{0/30} = 3^0 = 1$

$\therefore\ B(0) = 100 \cdot \underbrace{3^{0/30}}_{1} = 100$

as needed

bacteria TRIPLES means base is 3

(see explanation on exponent for more detail)

any base to the exponent zero is equal to 1

b) $t = 4$ days
$= 4 \text{ days} \times \frac{24\text{ h}}{1\text{ day}}$
$= 96\text{h}$

$\therefore B(96) = 100(3)^{\frac{(96)}{30}}$ ← on graphing calc: 100(3^(96/30))
$= 336\ 347$

So there will be about 336 347 bacteria in 4 days
(In Science class, we'd round this to 1 significant figure: 300 000 bacteria — why?)

c) If positive time is the future, then negative time is the past.

$t = -12\text{ h}$

$\therefore B(-12) = 100(3)^{\frac{(-12)}{30}}$ ← on graphing calc: 100(3^(-12/30))
≈ 64.4394015

So there was about 64 bacteria 12 hours ago.

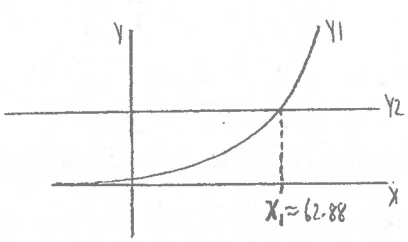
d) We want $B(t) = 1000$.

$\therefore \frac{1000}{100} = \frac{100(3)^{\frac{t}{30}}}{100}$

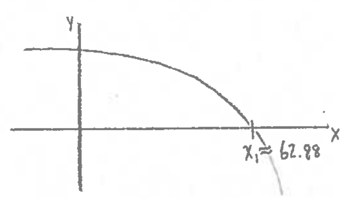
$10 = 3^{t/30}$ ← we cannot find t algebraically with the tools we've learned about so far (We will talk about this in ch 8)

There are several techniques you could use:
i) guess & check (i.e. trial + error)
ii) graph $y = (3)^{x/30}$ & read what x value corresponds to $y = 10$
iii) use graphing calculator as we've done in the past...

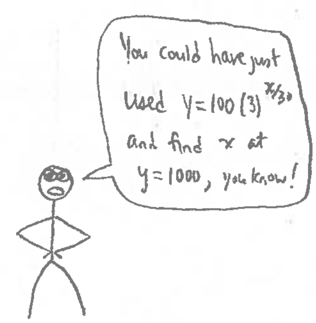
method I
Y1 = 3^(X/30)
Y2 = 10
use $\boxed{\text{INTERSECT}}$ function



method II
Y1 = 10 − 3^(X/30)
use $\boxed{\text{ZERO}}$ function



In any case, it takes about 63 days to get 1000 bacteria.



HWK: pg 342 − 344 # 3, 4, 8−12