Day 22 **Review:**

① Write an exponential equation to model each situation. Then answer the question.

a) A radioactive substance has a half-life of 8.2 days. After how long will only 25% of the substance be left?

b) The intensity of light below the surface of a lake is reduced by 5% for every metre below the surface. What percent of the original intensity remains 8m below the surface?

Sol^n: ① let $A(t)$ represent the amount of radioactive substance left after t days.

Then...

$$A(t) = 1\left(\frac{1}{2}\right)^{\frac{t}{8.2}}$$

we begin with 100%. As a decimal, 100% is 1

Substance is reduced by $\frac{1}{2}$ every 8.2 days

When $t = 8.2$, we expect $\frac{1}{2}$ of substance to remain:

$$\left(\frac{1}{2}\right)^{\frac{8.2}{8.2}} = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

and when $t = 8.2 \times 2 = 16.4$, we expect $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$ to remain: $\left(\frac{1}{2}\right)^{\frac{16.4}{8.2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

So $\frac{t}{8.2}$ works well as the exponent of $\frac{1}{2}$.

Now, we need to find t when $A(t)$ is 25% of its original amount.

If original amount is 100% or 1, then $A(t) = 25\% = 0.25$

$$\therefore\ 0.25 = \left(\frac{1}{2}\right)^{t/8.2}$$

$$0.25 = \frac{1}{4} = \frac{1}{2^2} = \left(\frac{1}{2}\right)^2 \longrightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/8.2} \Longrightarrow 2 = \frac{t}{8.2}$$

We want to express 0.25 with base $\frac{1}{2}$ because if we have like bases, then we can equate exponents ☺

Bases are the same, so the exponents must be equal for this equation to be true!

$$\therefore t = 16.4 \text{ days}$$

Thus, after 16.4 days, 25% of the substance will be left.

② let $I(d)$ represent the intensity of light at a depth of d metres below the surface of the lake.

Then...

$$I(d) = (0.95)^{d}$$

for every metre below, we get 0.95 of previous amount, so this works — confirm for $d = 0, 1, 2$ ☺

Begin at 100% & reduce by 5%, so: $100\% - 5\% = 95\% = 0.95$

Need to find $I(d)$ when $d = 8$ m:

$$I(8) = (0.95)^8 \approx 0.6634$$

Thus, about 66% of the original intensity remains 8m below the surface.

7.2 Transformations of Exponential Equations

There is nothing new here. You learned how to graph exponential functions of the form $y = b^x$ last section, and you have been applying transformations to functions throughout this course.

But, to be thorough (or just in case ☺):

wrt $y = b^x$...

base function

$$y = A(b)^{B(x+C)} + D$$

vertical translation: $D<0 \Rightarrow D$ units down
$D>0 \Rightarrow D$ units up

vertical stretch of factor $|A|$

horizontal stretch of factor $\left|\frac{1}{B}\right|$

horizontal translation:
$C<0 \Rightarrow C$ units right
$C>0 \Rightarrow C$ units left

Ex 1 Consider the function $y = 3^{-2(x+1)} - 4$

a) Describe the characteristics of its graph: domain, range, intercepts, asymptotes

b) graph the function using transformations on its base function.

Sol'n: a) It's often easier to graph and get this info that way — use graphing calculator ☺

Domain: All real numbers

Range: range for $y = b^x$ is $(0, \infty)$

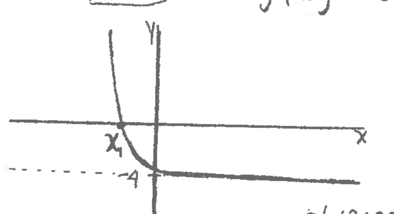
So use transformations!

$y = b^x - 4$ means range is $(-4, \infty)$

x-intercept: so $y = 0 \Rightarrow 0 = 3^{-2(x+1)} - 4$

so let Y1 = 3^(-2(X+1)) − 4

& use $\boxed{\text{ZERO}}$ Fn on graphing calc



$x_1 \doteq -1.63093$

∴ x-int is about −1.63

y-intercept: so $x = 0 \Rightarrow y = 3^{-2(0+1)} - 4$

$$= 3^{-2} - 4$$
$$= \frac{1}{3^2} - 4$$
$$= \frac{1}{9} - \frac{4}{1}\left(\frac{9}{9}\right)$$
$$= \frac{1}{9} - \frac{36}{9}$$
$$= -\frac{35}{9} \text{ or } -3\tfrac{8}{9}$$

∴ y-int is $-\frac{35}{9}$

asymptotes: we can see by the graph or eqn that the horizontal asymptote is $y = -4$

$$y = 3^{-2(x+1)} - 4$$

Note: not all exponential Fns have an x-int — why not?
Under what conditions would an x-int exist?

b) Use your transformations to go from base function $y=3^x$ to $y=3^{-2(x+1)}-4$

Recall mapping notation: $y=3^x \longrightarrow y=3^{-2(x+1)}-4$

$$(x,y) \longrightarrow \left(-\tfrac{1}{2}x-1,\ y-4\right)$$

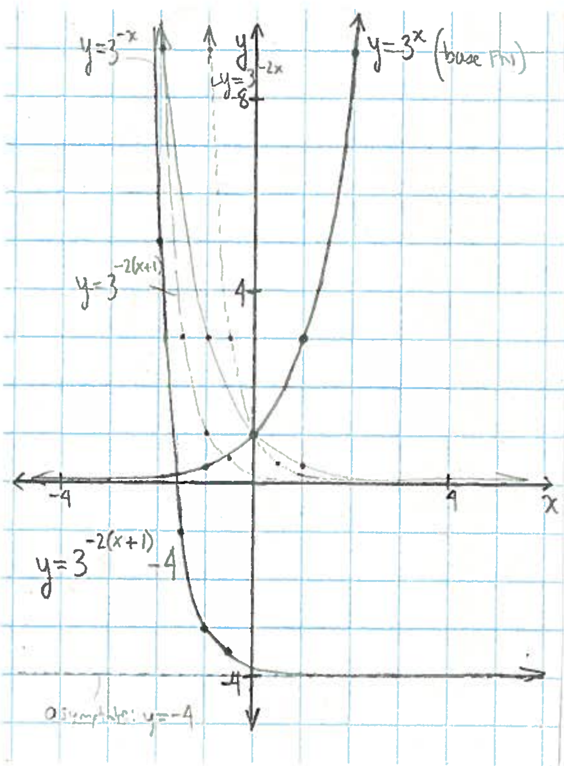
OR JUST perform the transformation right on the graph ☺ (see below)

x	y
-1	$\frac{1}{3}$
0	1
1	3
2	9

$\longrightarrow$

x	y	
$-\frac{1}{2}(-1)-1=-\frac{1}{2}$	$\left(\frac{1}{3}\right)-4=-3\frac{2}{3}$	$\rightarrow \left(-\frac{1}{2},-3\frac{2}{3}\right)$
$-\frac{1}{2}(0)-1=-1$	$(1)-4=-3$	$\rightarrow (-1,-3)$
$-\frac{1}{2}(1)-1=-1\frac{1}{2}$	$(3)-4=-1$	$\rightarrow \left(-1\frac{1}{2},-1\right)$
$-\frac{1}{2}(2)-1=-2$	$(9)-4=5$	$\rightarrow (-2,5)$

plot and join with smooth curve (see graph, below left)



– OR –

Make your life easier and simplify the function using properties of exponents...

$$y=3^{-2(x+1)}-4$$
$$=\left(3^{-2}\right)^{x+1}-4$$
$$=\left(\frac{1}{3^2}\right)^{x+1}-4$$
$$=\left(\frac{1}{9}\right)^{x+1}-4$$

fewer transformations to apply!

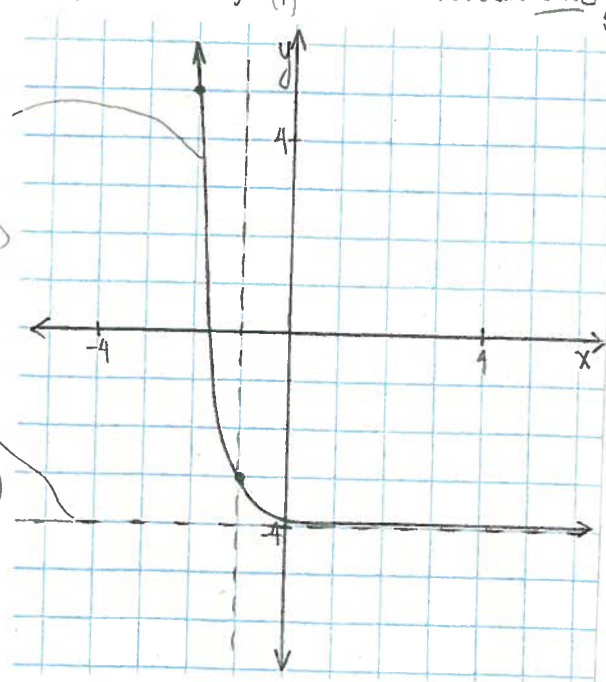
new base FN: $y=\left(\frac{1}{9}\right)^x$

Graph base FN $y=\left(\frac{1}{9}\right)^x$ and then translate axes!! (see below)

Originally $y=\left(\frac{1}{9}\right)^x$

Now it's $y=\left(\frac{1}{9}\right)^{x+1}-4$ with new axes

equivalent to $y=3^{-2(x+1)}-4$



original axes (x-axis becomes asymptote)

HWK: p 355–356

#4, 5c,d; 7d, 11, 12