Day 23 Review

① The formula for compound interest is given by $A(t) = P\left(1 + \frac{i}{n}\right)^{nt}$ where $A(t)$ is the amount of the investment after t years, P is the principal (initial amount), i is the annual interest rate (always a decimal), and n is the number of compounding periods per year. Determine the amount of interest \$500 earns, invested over a 10 year period at 6% per annum, compounded monthly.

② Describe how the graph of $y = 4^{-0.5x+3} + 1$ can be obtained by transforming the graph of $y = 4^x$.

Soln: ① $A(t) = ?$

$P = \$500$

$i = 6\% = 0.06$

$n = 12$ months/year

$t = 10$ years

$A(10) = 500\left(1 + \frac{0.06}{12}\right)^{12 \cdot 10}$

$= 500(1.005)^{120}$

$\approx 500(1.819396\ldots)$

$= \$909.70$

Need Interest:

$\$909.70 - \$500 = \$409.70$

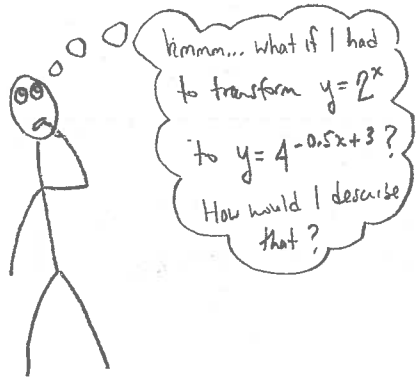
$\therefore$ \$409.70 in interest is earned.

② $y = 4^{-0.5x+3} + 1 \longrightarrow y = 4^{-0.5(x-6)} + 1$

- reflection in y-axis
- horizontal expansion about y-axis by factor 2
- horizontal translation 6 units right
- vertical translation 1 unit up

multiplicative transformations FIRST

additive transformations LAST



7.3 Solving Exponential Equations

Remember your exponent laws...

$$m^a \times m^b = m^{a+b}$$

$$m^a \div m^b = m^{a-b}$$

$$(m^a)^b = m^{ab}$$

Can verify easily — see below!

$$m^{\frac{a}{b}} = \left(\sqrt[b]{m}\right)^a$$

$$m^{\frac{1}{b}} = \sqrt[b]{m}$$

$$m^{-a} = \frac{1}{m^a}$$

Using patterns, these definitions make sense — look it up ☺

Ex 1 Write each expression as a power with base 2.

a) 4^3 b) $\frac{1}{8}$ c) $8^{\frac{2}{3}}\left(\sqrt{16}\right)^3$

Soln: a) $4^3 = (2^2)^3$
$= 2^{2\cdot 3}$
$= 2^6$

b) $\frac{1}{8} = \frac{1}{2^3}$
$= 2^{-3}$

c) $8^{\frac{2}{3}}\left(\sqrt{16}\right)^3 = (2^3)^{\frac{2}{3}}(4)^3$
$= 2^{3\left(\frac{2}{3}\right)}(2^2)^3$
$= 2^2 \cdot 2^6$
$= 2^{2+6}$
$= 2^8$

ex $2^2 \cdot 2^6 = (2\cdot 2)(2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2)$
$= 2^8$
So I add $2+6=8$ ✓

ex $(2^2)^3 = (2\cdot 2)^3$
$= (2\cdot 2)(2\cdot 2)(2\cdot 2)$
$= 2^6$
so I multiply $2\times 3 = 6$ ✓

TRY FOR $\frac{2^5}{2^3}$ ...

Ex 2: Solve each equation:

a) $8^{3x-2} = 16^{x+1}$ b) $27^{x+3} = \left(\frac{1}{9}\right)^{2x-5}$ c) $\frac{8^{x+6}}{16^{2x-1}} = 32^{3x-4}$

* The idea is to get powers of the same base so that you can equate exponents — why?

Consider this... if $2^3 = 2^x$, then the only way this statement is true is when $x = 3$

(we just equated exponents when bases are the same!)

Soln: a) $8^{3x-2} = 16^{x+1}$

$(2^3)^{3x-2} = (2^4)^{x+1}$

$2^{3(3x-2)} = 2^{4(x+1)}$

$2^{9x-6} = 2^{4x+4}$

like bases, so equate exponents...

$\therefore 9x - 6 = 4x + 4$

$-4x \quad -4x$

$5x - 6 = 4$

$+6 \quad +6$

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

b) $27^{x+3} = \left(\frac{1}{9}\right)^{2x-5}$

$(3^3)^{x+3} = \left(\frac{1}{3^2}\right)^{2x-5}$

$3^{3(x+3)} = (3^{-2})^{2x-5}$

$3^{3x+9} = 3^{-2(2x-5)}$

$3^{3x+9} = 3^{-4x+10}$

$\therefore 3x + 9 = -4x + 10$

$7x = 1$

$x = \frac{1}{7}$

c) $\frac{8^{x+6}}{16^{2x-1}} = 32^{3x-4}$

$\frac{2^{3(x+6)}}{2^{4(2x-1)}} = 2^{5(3x-4)}$

$2^{3(x+6)-4(2x-1)} = 2^{5(3x-4)}$

$2^{3x+18-8x+4} = 2^{15x-20}$

$2^{-5x+22} = 2^{15x-20}$

$\therefore -5x + 22 = 15x - 20$

$42 = 20x$

$\frac{42}{20} = x$

$\therefore x = \frac{21}{10}$ or 2.1

HWK: p364-365 #5, 11, 14