Day 27 Review

① Solve for x. Give the exact value.

a) $5^{2x-1} = 2(7^{3-x})$ b) $\log_2(x-2) + \log_2(x-5) = 2$

② pg 414 # 12 a) Using eqn: $\log T = \frac{3}{2}\log d - 3.263$, where T is time in Earth years for a planet to revolve around the sun and d is its distance, in millions of km, from the sun. Pluto is on average 5906 million km from the sun. How long does Pluto take to orbit the sun?

③ The average car depreciates 15% per year. In addition to this, the car immediately depreciates about 20% just for having purchased it at a retail rather than its wholesale price. Write an equation to model the resale price, $P(t)$, after t years if the initial retail price of the vehicle is \$19,950. Then determine how long it would take to depreciate to a resale price of \$10,000.

Soln: ① a) $5^{2x-1} = 2(7^{3-x})$

$\log(5^{2x-1}) = \log[2(7^{3-x})]$

$(2x-1)\log 5 = \log 2 + \log(7^{3-x})$

$(2\log 5)x - \log 5 = \log 2 + (3-x)\log 7$

$(2\log 5)x - \log 5 = \log 2 + 3\log 7 - (\log 7)x$

$(2\log 5)x + (\log 7)x = \log 2 + 3\log 7 + \log 5$

$(2\log 5 + \log 7)x = \log 2 + \log 5 + 3\log 7$

$x = \dfrac{\log 2 + \log 5 + 3\log 7}{2\log 5 + \log 7}$

b) $\log_2(x-2) + \log_2(x-5) = 2$

$\log_2[(x-2)(x-5)] = 2$

exp form: $(x-2)(x-5) = 2^2$

$x^2 - 7x + 10 = 4$

$x^2 - 7x + 6 = 0$

$(x-1)(x-6) = 0$

$\therefore x = 1$ or $x = 6$

check restrictions: $\log_2(\underbrace{x-2}_{>0})$ & $\log_2(\underbrace{x-5}_{>0})$

$\therefore x > 2$ $\therefore x > 5$

$\therefore x > 5$

So $x \neq 1$ and $x = 6$ is only solution

② $\log T = \frac{3}{2}\log d - 3.263$

$d = 5906$ million km

$T = ?$ (earth years)

$\log T = \frac{3}{2}\log 5906 - 3.263$

exp form: $T = 10^{\frac{3}{2}\log 5906 - 3.263}$

≈ 247.7080748

$\therefore$ Takes Pluto about 248 Earth years to orbit the sun.

③ Assume car depreciates 15% per year based on its initial price.

Then... $P(t) = \underbrace{19950(0.85)^t}_{\text{annual depreciation of 15\% based on original price}} - \underbrace{19950(0.2)}_{\text{initial depreciation of 20\%}}$

Solve for t when... $10000 = 19950(0.85)^t - 3990$

$13990 = 19950(0.85)^t$

$0.7012531328 \approx (0.85)^t$

log of both sides... $t \approx 2.18$

$\log(0.70125) \approx \log 0.85^t$

$\frac{\log(0.70125)}{\log .85} \approx \frac{t\log 0.85}{\log 0.85}$

$t \approx \frac{\log 0.70125}{\log 0.85}$

$\therefore$ It takes about 2.2 years to depreciate to \$10000.

CHAPTER 11 – Permutations, Combinations, Binomial Theorem

11.1 – Permutations (I)

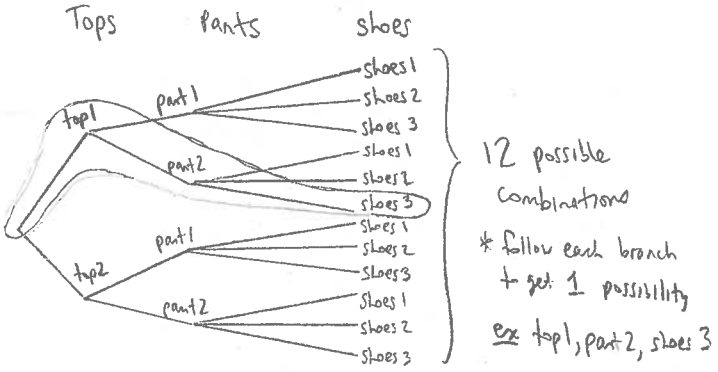
The Fundamental Counting Principle

Ex 1 How many unique outfits can you make with:

- 2 different tops
- 2 different pants
- 3 different pairs of shoes

Assume you must wear one from each type of item above.

method 1: Tree diagram



12 possible combinations

* follow each branch to get 1 possibility

ex top1, pant2, shoes 3

method 2: Fundamental Counting Principle

We can see from the tree diagram that $2 \times 2 \times 3$ leads to the number of possibilities:

$$\underset{\#\text{tops}}{2} \times \underset{\#\text{pants}}{2} \times \underset{\#\text{shoes}}{3} = 12 \text{ ways}$$

(For each top there are 2 pants; and for each of those combinations there are 3 shoes)

Fundamental Counting Principle: If one task can be performed in a ways and another task can be performed in b ways, then the two tasks can be performed in $a \times b$ ways. This can be generalized for any number of tasks.

Ex 2: How many licence plates can be made consisting of 3 digits followed by 3 letters? What if no digits or letters are repeated?

$$\underset{0\text{ to }9}{10} \times \underset{0\text{ to }9}{10} \times \underset{0\text{ to }9}{10} \times \underset{A\text{ to }Z}{26} \times \underset{A\text{ to }Z}{26} \times \underset{A\text{ to }Z}{26} = 17\,576\,000 \text{ possible licence plates}$$

without repetition: $\underset{0\text{ to }9}{10} \times \underset{\substack{1\text{ less to choose from}}}{9} \times \underset{2\text{ less}}{8} \times \underset{A\text{ to }Z}{26} \times \underset{\substack{1\text{ less to choose from}}}{25} \times \underset{2\text{ less}}{24} = 11\,232\,000$ possible licence plates

Factorial Notation

Ex 3 How many different ways can we order 5 people in a line?

$$\underset{\text{1st person (5 choices)}}{5} \times \underset{\text{2nd person (4 choices left)}}{4} \times \underset{\text{3rd (3 left)}}{3} \times \underset{\text{4th (2 left)}}{2} \times \underset{\text{5th (1 left)}}{1} = 120 \text{ ways}$$

The above example of descending consecutive numbers written as a product is a familiar arrangement — so familiar that it can be abbreviated as: $5!$

That is, $5! = 5 \times 4 \times 3 \times 2 \times 1$ (we say "five factorial")

And in general, $n! = n(n-1)(n-2)\cdots(3)(2)(1)$, where $n \in \mathbb{N}$

* note: $0! = 1$ by definition

On graphing calc: MATH ▸ PRB ▸ 4:!

The above arrangement is also called a permutation; that is, an arrangements of objects where order is important.

Ex 4 a) $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 40\,320$

b) $\frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$ ($3! = 3 \times 2 \times 1$ we leave it like $3!$ to divide with denominator ☺)
$= 7 \times 6 \times 5 \times 4$
$= 840$

Permutations

In general we use ${}_nP_r$ to represent the number of **permutations** (arrangements in a definite order) of r items taken from a set of n distinct items.

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}} \quad \text{where } n \in \mathbb{N} \quad n \geq r$$

Ex 5 How many ways can we hang 3 different paintings in a line on a wall from a choice of 5 paintings?

$\underset{\text{1st}}{5} \times \underset{\text{2nd}}{4} \times \underset{\text{3rd}}{3} = 60$ ways

-OR- use ${}_nP_r = \frac{n!}{(n-r)!}$

$n = 5$, $r = 3$ $\therefore {}_5P_3 = \frac{5!}{(5-3)!}$
$= \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$
$= 60$

(can also do on graphing calc *)

* On graphing calc: 5 MATH ▸ PRB ▸ 2: ${}_nP_r$ 3

Ex 6 Show that $100! + 99! = 101(99!)$

$LS = 100! + 99!$

$= 100 \times 99! + 99!$

$= (100 + 1)\,99!$ (factor out $99!$)

$= (101)\,99!$

$= RS$ $\therefore\ 100! + 99! = 101(99!)$

Ex 7 Solve for n if ${}_nP_2 = 56$.

${}_nP_2 = \dfrac{n!}{(n-2)!}$ and ${}_nP_2 = 56$

$\therefore\ \dfrac{n!}{(n-2)!} = 56$

$\dfrac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 56$

$n(n-1) = 56$

$n^2 - n - 56 = 0$

$(n-8)(n+7) = 0$

$\therefore\ n = +8$ or ~~$n = -7$~~ extraneous b/c $n \in \mathbb{N}$

$\therefore\ n = 8$ is the only solution

Check on calc! ${}_8P_2 = 56$ ✓

HWK: p 524 – 526 # 4e, 6, 7a, 13, 14, 15, 20, 21, 22c

(With combinatorics, I find the more questions you can reason through, the better! Do as many as you can ☺)