Day 28 Review

① A family of 2 adults and 3 children are lining up for a family portrait.

a) How many different ways can they order themselves in a line?

b) How many different ways can they order themselves if an adult must be at each end of the line?

c) How many possible ways can any three be ordered in a line?

d) How many possible ways can 2 or 3 family members be ordered in a line?

② Solve algebraically: $n! = 12(n-2)!$

Sol'n: ① a) $n=5$ $\quad 5! = 120$ ways $\quad$ -or- $\quad \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$

b) $\underset{A}{\underline{2}} \times \underset{C}{\underline{3}} \times \underset{C}{\underline{2}} \times \underset{C}{\underline{1}} \times \underset{A}{\underline{1}} = 12$ ways

c) $n=5$, $r=3$ $\quad {}_5P_3 = \underbrace{\frac{5!}{(5-3)!}}_{\text{not necessary if you use nPr on calc ☺}} = 60$ ways $\quad$ -or- $\quad \underline{5} \times \underline{4} \times \underline{3} = 60$

d) The "or" introduces 2 cases:

i) $n=5$ with $r=2 \longrightarrow {}_5P_2 = 20$

ii) $n=5$ with $r=3 \longrightarrow {}_5P_3 = 60$

always ADD cases

80 ways

② $n! = 12(n-2)!$

$n(n-1)\cancel{(n-2)!} = 12\cancel{(n-2)!}$

$n^2 - n = 12$

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$\therefore n = 4,\ \cancel{n=-3}$ not possible b/c $n \in \mathbb{N}$

$\therefore n = 4$ only sol'n

Check: $LS = 4! = 24$ $\quad RS = 12(4-2)! = 12(2)! = 24$

$\because LS = RS \therefore n = 4$ ✓

11.1 Permutations (II)

Permutations with repeating objects

Consider the number of 3-letter arrangements possible in the word "mom":

mom   mmo   omm   omm   mom   mmo   ($3! = 6$ arrangements)

Notice that the two identical letters (m) give us repeated arrangements which we need not include. Eliminating repeats gives:

mom   mmo   omm   ($\frac{3!}{2!} = 3$ unique arrangements)

In general, the number of unique arrangements of n objects of which a objects are identical, another b objects are identical, another c objects are identical, etc. is given by:

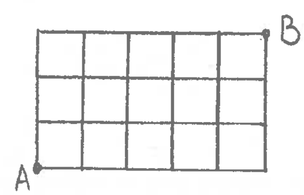
$$\frac{n!}{a!\,b!\,c!\cdots}$$

Ex1 How many ways could you uniquely arrange the letters in the word "MISSISSAUGA"?

sol $n = 11$
$S = 4$
$i = 2$
$a = 2$

$\therefore \frac{11!}{4!\,2!\,2!} = 415\,800$ ways

Ex2 How many ways can you get from point A to B on the grid by only moving up or right?



sol No matter which path you choose, you'll go up (U) three times and right (R) five times

UUU RRRRR

So $n = 8$
$U = 3$
$R = 5$

$\therefore \frac{8!}{3!\,5!} = 56$ ways

More challenging examples of permutations

Often constraints or different cases to consider can add a new challenge to what was once a familiar question!

Ex 3 Five people are seated on a bench. In how many ways can they be arranged if:

a) There is a couple who must sit together?

b) The couple cannot sit together?

c) There are 3 people who must sit together at one end?

Sol^n: For convenience, let's call the people A, B, C, D, E

a) Say the couple is A & B. They must sit together and can do so in $2!$ ways: AB or BA.

Since they must stay together, let's treat them as one: AB, C, D, E

These 4 can be arranged $4!$ ways. Therefore they can be arranged $2!4! = 48$ ways

visually: $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times 2$ since we can switch A & B, giving 48 ways.

b) Without restrictions, 5 people can sit $5!$ ways visually: $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

From a), number of ways the couple can sit together is $2!4!$.

Therefore, the number of ways the couple is not sitting together is:

$$5! - 2!4! = 120 - 48 = 72$$

c) Let's call these 3 people A, B, C. They can sit together in $3!$ ways.

If they are together, treat them as one: ABC, D, E

Two cases: $3! \times \underset{ABC}{\underline{1}} \times \underset{\text{2 left to choose}}{\underline{2}} \times \underset{\text{1 left}}{\underline{1}}$ or $\underset{\text{2 left to choose}}{\underline{2}} \times \underset{\text{1 left}}{\underline{1}} \times \underset{ABC}{\underline{1}} \times 3!$

$$= 3!2! + 3!2!$$
$$= 24 \text{ ways}$$

-or- think... $3!2! \times \underset{\text{to be on}}{2 \text{ ends}} = 24$ ways

Ex 4 How many different 3-digit even numbers greater than 300 can you make using the digits 1, 2, 3, 4, 5, 6? No digits are repeated.

Sol'n: There are 2 cases to consider:

6 digits − 2 digits chosen for 1st + 3rd digit

① Beginning with 3 or 5 : $\underset{\text{3 or 5}}{\underline{2}} \times \underset{\text{the rest (fill in last)}}{\underline{4}} \times \underset{\text{even } (2,4,6)}{\underline{3}} = 24$

② Beginning with 4 or 6 : $\underset{\text{4 or 6}}{\underline{2}} \times \underset{\text{the rest}}{\underline{4}} \times \underset{\text{even (2 left to choose from)}}{\underline{2}} = 16$

add cases: 40

So we can make 40 3-digit even numbers greater than 300.

HWK: pg 524 # 5c, 8c, 10–12, 19, 25–28

challenge... try some of the "extend" questions ☺