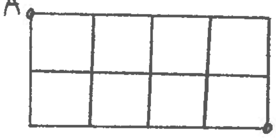
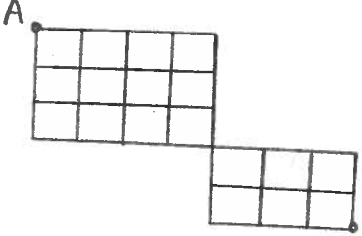
Day 29 Review

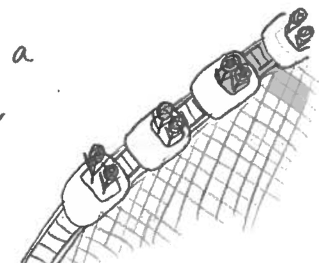
① How many ways can you get from point A to point B on the grid by only moving down or right?

a) 

b) 

(# 27 p 526) ② How many integers between 1 and 1000 do not contain repeated digits?

③ Harold and Maude and three of their friends are going on a rollercoaster with a train holding two people per car. How many unique ways can they sit in the first three cars if Harold and Maude must sit together?

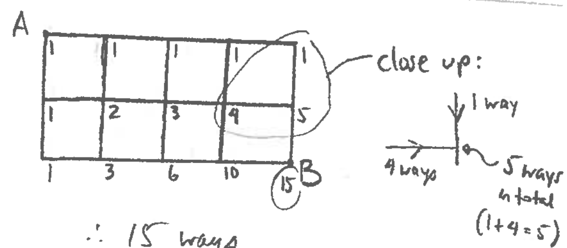


Sol^n ① a) method 1: DDRRRR

$n = 6$
$D = 2$
$R = 4$

$\therefore \dfrac{6!}{2!\,4!} = 15$ ways

method 2:



$\therefore$ 15 ways

b) If you try to do it like this: ___ ___ ___ you run into issues.

Split into 3 cases: 3 digit 2 digit 1 digit

don't include 1 + 1000

$\underset{1 \text{ to } 9}{9} \times \underset{\substack{0 \text{ to } 9\\ -1}}{9} \times \underset{\substack{0 \text{ to } 9\\ -2}}{8} + \underset{1 \text{ to } 9}{9} \times \underset{\substack{0 \text{ to } 9\\ -1}}{9} + \underset{2 \text{ to } 9}{8} = 737$ integers

used here used here (used here)

c) Treat Harold & Maude as 1 but realize they can be arranged 2! ways and still be together. So then we have 3 cases:

H&M in 1st car: $\underbrace{2!}_{H \& M} \times \underbrace{4 \times 3}_{2^{nd} \text{ car}} \times \underbrace{2 \times 1}_{3^{rd} \text{ car}} = 48$

H&M in 2nd car: $4 \times 3 \times 2! \times 2 \times 1 = 48$

H&M in 3rd car: $4 \times 3 \times 2 \times 1 \times 2! = 48$

144 ways

(Count "extra" seat as another, 6th rider (or empty seat assigned) $\Rightarrow$ 4 spaces to fill)

$\longrightarrow$ or simply: $3 \times 2!\,4! = 144$ ☺

11.2 - Combinations

A combination differs from a permutation in that order is not important.

i.e. a <u>Combination</u> is a selection of objects without regard to order

Like a "perm", a "com" can be found using the Fundamental Counting Principle ... but we must remember to divide out the number of ways an object can be arranged (i.e. eliminate repetitions based on order)

Ex 1 How many ways can we choose 3 people from a group of 5?

Using Fundamental Counting Principle:

$$\underline{5} \times \underline{4} \times \underline{3} = 60$$

but order doesn't matter, so divide by possible arrangements of 3 objects: $3!$

ie $\frac{60}{3!} = \frac{60}{6} = 10$

That is ...

$$\# \text{coms} = \frac{\# \text{perms}}{r!} \qquad \text{where } r \text{ is } \# \text{ chosen}$$

$$= \frac{60}{3!}$$

$$= \frac{60}{6}$$

$$= 10$$

This leads to the following formula ...

The number of combinations of n items taken r at a time is given by:

$$\boxed{{}_nC_r = \frac{{}_nP_r}{r!}} \quad \text{or} \quad \boxed{{}_nC_r = \frac{n!}{(n-r)!\,r!}}$$

where $n \geq r \geq 0$ & $n \in \mathbb{N}$

Note: $\binom{n}{r}$ is another notation for ${}_nC_r$

<u>On graphing calculator:</u>

"5 choose 3" is common lingo ☺

For above example, we'd write: ${}_5C_3 = \frac{{}_5P_3}{3!} = \frac{5!}{(5-3)!\,3!} = 60.$

demonstrated by $\boxed{5}$ $\boxed{\text{MATH}}$ ▷ PRB ▽ 3: nCr $\boxed{3}$ on TI-83 (& above)

Ex 2 There are 8 males and 6 females in a Pre-Calculus 12 class. The teacher needs to choose 5 of them to help with a problem.

a) How many selections are possible?

b) How many selections are possible containing exactly 3 females?

c) How many selections are possible containing at least 1 male?

Soln: a) $n = 8 + 6 = 14$ $\quad \therefore {}_{14}C_5 = 2002$ possible selections

$r = 5$

b) Exactly 3 females means exactly 2 males also:

males: females:

$${}_8C_2 \times {}_6C_3 = 560 \text{ possible selections}$$

(choose 2 males from 8) (choose 3 females from 6)

multiply to get all combinations — not separate cases!

c) "At least" implies cases: 1 male, 2 males, 3 males, 4 males, 5 males.

It's easier to consider all possible (unrestricted) selections and subtract the case of Ø males ☺

<u>method 1 (quicker):</u>

$${}_{14}C_5 - \overset{1}{{}_8C_0} \cdot {}_6C_5 = 2002 - 6 = 1996 \text{ possible selections}$$

(no restrictions) (no ♂) (5 ♀)

know this from a) so even easier !!

<u>method 2 (longer):</u>

$${}_8C_1 \times {}_6C_4 + {}_8C_2 \times {}_6C_3 + {}_8C_3 \times {}_6C_4 + {}_8C_4 \times {}_6C_1 + {}_8C_5 \times {}_6C_0$$

why is this factor not necessary to write?

$= 120 + 560 + 840 + 420 + 56$

$= 1996$ possible selections

Sum to n Sum to r

Ex 3 Solve for n if $720\left({}_nC_5\right) = {}_{n+1}P_5$

sol

$$\overset{6}{\cancel{720}}\left(\frac{n!}{(n-5)!\,\underset{1}{\cancel{5!}}}\right) = \frac{(n+1)!}{((n+1)-5)!}$$

$$\frac{6n!}{(n-5)!} = \frac{(n+1)!}{(n-4)!}$$

$$\frac{6\cancel{n!}}{\cancel{(n-5)!}} = \frac{(n+1)\cancel{n!}}{(n-4)\cancel{(n-5)!}}$$

divide $n!$ from both sides

divide $(n-5)!$ from both sides

$$\frac{6}{1} = \frac{n+1}{n-4}$$

$$6(n-4) = n+1$$

multiplied both sides by $(n-4)$

$$6n-24 = n+1$$

$$-n \quad +24 \qquad -n \quad +24$$

$$5n = 25$$

$$\boxed{n = 5}$$

aside:

$$(n+1)! = (n+1)((n+1)-1)((n+1)-2)((n+1)-3)\cdots$$
$$= (n+1)(n)(n-1)(n-2)\cdots$$
$$= (n+1)\,n!$$

$$(n-4)! = (n-4)((n-4)-1)((n-4)-2)\cdots$$
$$= (n-4)(n-5)(n-6)\cdots$$
$$= (n-4)(n-5)!$$

HWK: pg 534-536 # 6c, 9–13, 15–21