Day 3

Review: ① Write an equation that represents the graph of $y=f(x)$ after:

a) it is translated 2 units right and 4 units down

b) it is stretched horizontally by a factor of 5.

② Write an equation that represents $y=x^3+x^2$ after it is reflected in:

a) the x-axis

b) the y-axis

---

soln: ① a) $y=f(x-2)-4$

b) $y=f\left(\frac{1}{5}x\right)$ or $y=f\left(\frac{x}{5}\right)$

② a) reflection in x-axis means y-values change sign (x-values stay the same)

$\therefore\ y=x^3+x^2 \longrightarrow (-y)=x^3+x^2$

So $y=-x^3-x^2$

b) reflection in y-axis means x-values change sign (y-values stay the same)

$\therefore\ y=x^3+x^2 \longrightarrow y=(-x)^3+(-x)^2$

So $y=-x^3+x^2$

Today's HWK: ☐ 1.3 pg 39 #6, 7, 9, 10, 11

☐ 1.4 pg 51 #1-5, 7, 9, 14

1.3 Combining Transformations

In general: given graph $y = f(x)$, $\quad y = a\,f\big(b(x-c)\big) + d$

represents the following transformations on $y = f(x)$:

Using mapping notation: $(x, y) \rightarrow \left(\frac{x}{b} + c,\ ay + d\right)$

$|a| > 1 \Rightarrow$ vertical expansion by factor a about x-axis
$|a| < 1 \Rightarrow$ vertical compression by factor a about x-axis
$a < 0 \Rightarrow$ reflection in x-axis

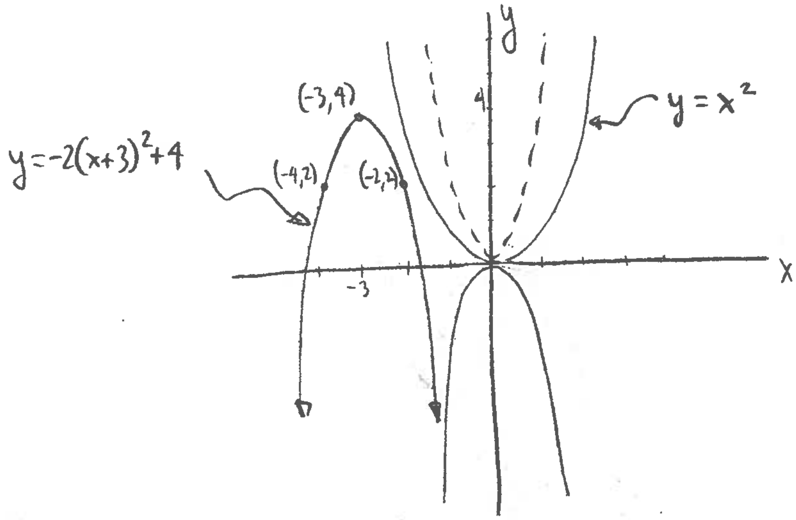
$|b| > 1 \Rightarrow$ horizontal compression by factor $\frac{1}{b}$ about y-axis
$|b| < 1 \Rightarrow$ horizontal expansion by factor $\frac{1}{b}$ about y-axis
$b < 0 \Rightarrow$ reflection in y-axis

$c > 0 \Rightarrow$ translation to right by c units
$c < 0 \Rightarrow$ translation to left by c units

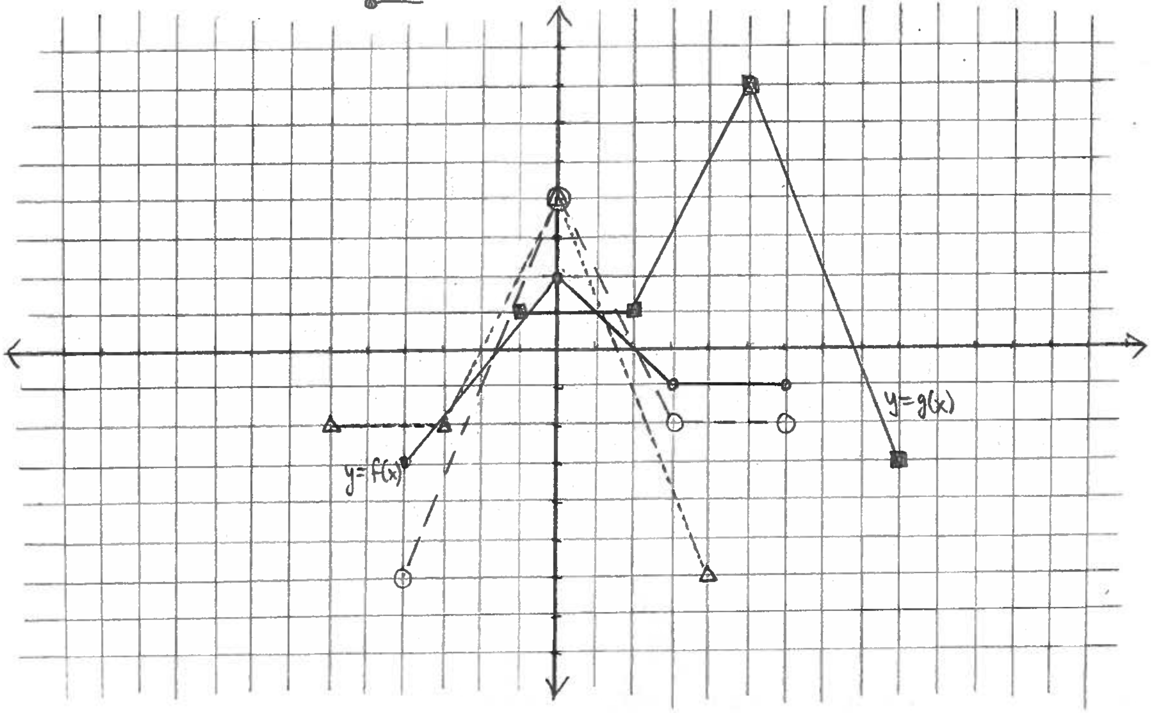
$d > 0 \Rightarrow$ translation up by d units
$d < 0 \Rightarrow$ translation down by d units

* perform multiplicative transformations first (i.e., a, b, including reflections)
perform additive transformations last (i.e. c, d)

Ex sketch $y = x^2$ and $y = -2(x+3)^2 + 4$ on the same graph.



Ex 2 The graph of $y = f(x)$ is shown below. Sketch $g(x) = 2f(5-x) + 3$



First put $g(x)$ in standard form to easily see transformations:

$$g(x) = 2f(-x+5) + 3$$

$$g(x) = 2f(-[x-5]) + 3$$

translation UP 3 units

--o vertical stretch of factor 2

---△ reflection in y-axis

translation RIGHT 5 units

often easiest to do together (less clutter)

* do multiplicative transformations first : stretches + reflections

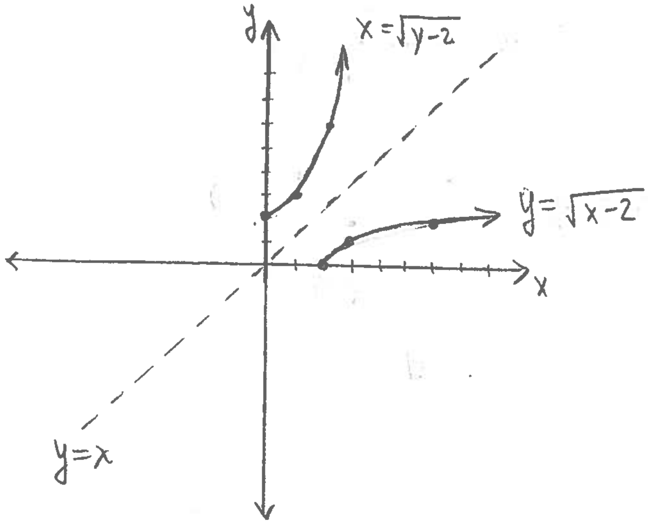
* do additive transformations last : translations

Using mapping notation, the transformation can be described as:

$$(x, y) \longrightarrow (-x+5,\ 2y+3)$$

Now moving on to 1.4 — Inverse of a Relation ...

Ex 1 Let $f(x) = \sqrt{x-2}$. Write an equation for the inverse of $f(x)$. Graph them on the same axes. How are they related? State Domain + Range of each.



$y = \sqrt{x-2}$

x	y
2	0
3	1
6	2

$x = \sqrt{y-2}$

x	y
0	2
1	3
2	6

Switch x + y values!

Let $y = f(x)$

To find the inverse, switch $x \longleftrightarrow y$ and solve for y:

$y = \sqrt{x-2}$ note: $\underset{\text{domain}}{x \geq 2}$; $\underset{\text{range}}{y \geq 0}$

Switch $x \leftrightarrow y$: $x = \sqrt{y-2}$ ← this is the inverse, but by convention we put it in "y =" form

$x^2 = y-2$

$y = x^2 + 2$ note: $y \geq 2$; $x \geq 0$ the opposite!!

Inverse of f(x), written as:

$f^{-1}(x) = x^2 + 2$, $\underbrace{y \geq 2 ; x \geq 0}$ restrictions b/c of original function

(sometimes seen as $x = f(y)$)

Graphically, we see that x and y values are interchanged.
The points along the $y=x$ line would thus be invariant. This is the mirror line.

So, $x = f(y)$ is the reflection of $y = f(x)$ in the line $y=x$.

$x = f(y)$ and $y = f(x)$ are inverses of one another.

We usually write $y = f(x)$ and its inverse $x = f(y)$ as $y = f^{-1}(x)$

ex $f(x) = \sqrt{x-2}$ + its inverse is $f^{-1}(x) = x^2-2$, $x \geq 0$.

* Domain + Range of relation become Range + Domain, respectively, of the inverse relation

Ex 2 Let $f(x) = x^2 + 1$

Could also be written as $y = f^{-1}(x)$

a) Write an equation for $y = -f(x)$, $y = f(-x)$, and $x = f(y)$

b) Sketch all four on the same axes.

c) How is the graph of each equation in a) related to the graph of $y = f(x)$?

d) Are there any invariant points?

e) Which equations represent functions? (vertical + horizontal line test)

Sol^n: a)

$y = -f(x)$
$= -(x^2+1)$
$= -x^2 - 1$
$\therefore y = -x^2 - 1$

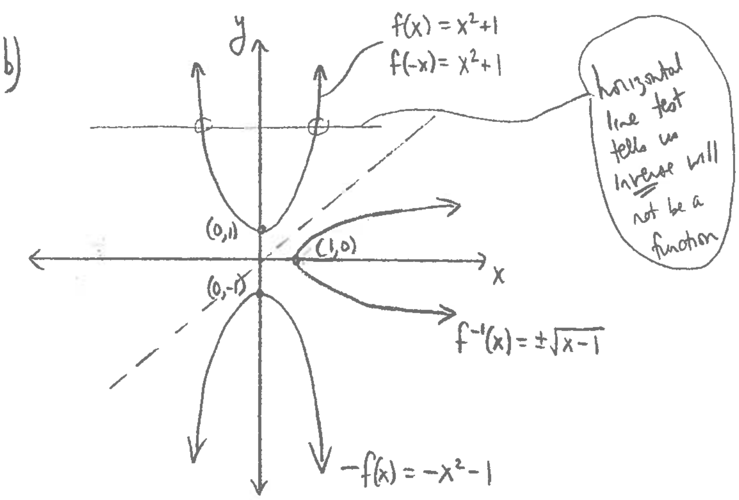
$y = f(-x)$
$= (-x)^2 + 1$
$= x^2 + 1$
$\therefore y = x^2 + 1$

$x = f(y)$
$x = y^2 + 1$
$x - 1 = y^2$
$\pm\sqrt{x-1} = y$
$\therefore y = \pm\sqrt{x-1}$ or $f^{-1}(x) = \pm\sqrt{x-1}$

note: taking square root of a variable with even exponent yields $\pm$

why?
ex if $y = 1$
$y^2 = 1$
if $y = -1$
still: $y^2 = 1$

b)



c) $y = -f(x)$ is $y = f(x)$ reflected in the x-axis
$y = f(-x)$ is $y = f(x)$ reflected in the y-axis
$x = f(y)$ is $y = f(x)$ reflected in the line $y = x$

d) The point $(0,1)$ is invariant when $y = f(x)$ is reflected in the y-axis (not all — the rest switched!)
ex $(1,2) \longleftrightarrow (-1,2)$
didn't stay the same

But because $f(x) = f(-x)$, $f(x)$ is an even function.
See pg 29 of textbook if interested ☺ [not on test!]

e) Use the vertical line test! It is clear that the only one that is not a function is:
$x = f(y)$ or $f^{-1}(x) = \pm\sqrt{x-1}$ because there are 2 y-values for at least 1 x-value

ex $x = 2$ gives $y = +1$ or $y = -1$