Day 4

☐ renew

① $(2,-2)$ is a point on the graph of $y=f(x)$. Determine a point on the graph of $y=-4f(6-2x)+10$.

② Determine the inverse of $y=-5x^2+3$. Is the inverse a function?

③ Sketch a graph of $y=2|4-x|-3$.

Sol'n: ① use mapping notation (fast!) ... but 1st put in proper form:

$$y=-4f(6-2x)+10$$
$$y=-4f(-2x+6)+10$$
$$y=-4f(-2(x-3))+10$$

a b c d

$$(x,y)\longrightarrow\left(-\frac{x}{2},-4y\right)$$
$$\left(-\frac{x}{2},-4y\right)\longrightarrow\left(-\frac{x}{2}+3,\,-4y+10\right)$$

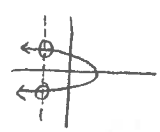
So $(2,-2)\rightarrow\left(-\frac{(2)}{2}+3,\,-4(-2)+10\right)=(2,18)$

② Switch $x\leftrightarrow y$

$$x=-5y^2+3$$
$$x-3=-5y^2$$
$$\frac{x-3}{-5}=y^2$$
$$\therefore\ y=\pm\sqrt{\frac{x-3}{-5}}$$

Inverse is NOT a function by...

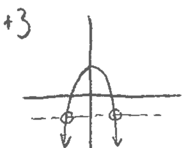
① vertical line test on $y=\pm\sqrt{\frac{x-3}{-5}}$ (checking inverse directly)



vertical line passes 2 or more pts

—OR—

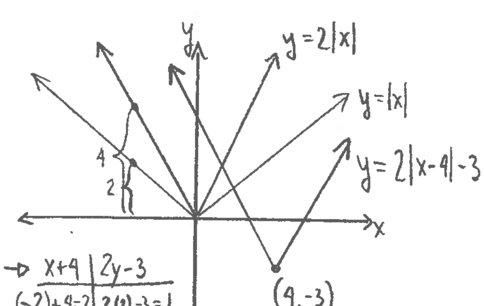
② horizontal line test on $y=-5x^2+3$ (checking inverse indirectly by testing original fn)



horizontal line passes 2 or more pts

③ First put in standard form: $y=2|4-x|-3=2|-x+4|-3=2|-(x-4)|-3$ $\therefore\ y=2|-(x-4)|-3$

note: $|-(x-4)|=|-1||x-4|$ $=|x-4|$ easier to graph



x	y	→	x+4	2y-3
-2	2		(-2)+4=2	2(2)-3=1
-1	1		(-1)+4=3	2(1)-3=-1
0	0		(0)+4=4	2(0)-3=-3
1	1		(1)+4=5	2(1)-3=-1
2	2		(2)+4=6	2(2)-3=1

Can use table of values:

$y=|x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

$y=2|x|$

x	y
-2	2(2)=4
-1	2
0	0
1	2
2	4

$y=2|x-4|-3$

x	y
-2+4=2	4-3=1
3	-1
4	-3
5	-1
6	1

So just $y=2|x-4|-3$

new vertex pt: $(4,-3)$

or mapping notation:

$$(x,y)\longrightarrow(x+4,\,2y-3)$$

(see table of values on left)

2.1 Radical Functions & Transformations

Ex1 Sketch the graph of $y=\sqrt{x}$ and $y+3=\sqrt{-(x+4)}$. Compare the domain and range of each graph. How can mapping notation help with graphing?

Sol'n: first: Put in standard form to graph more easily: $y=\sqrt{-(x+4)}-3$

method 1: graphically

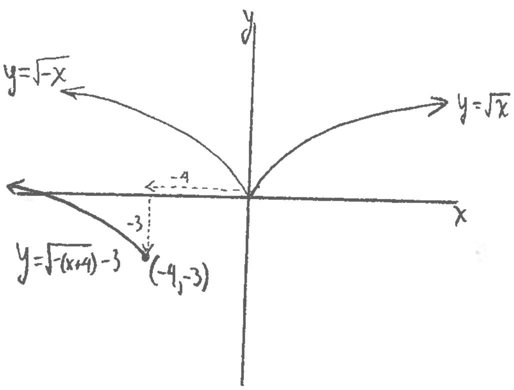
plot $y=\sqrt{x}$ — base function we are already familiar with (use table of values if needed)

then examine:

$$y=\sqrt{-(x+4)}-3$$

- reflection of $y=\sqrt{x}$ in y-axis (DO FIRST)
- shift $y=\sqrt{-x}$ left 4
- shift $y=\sqrt{-(x+4)}$ down 3

OR Can see $(-4,-3)$ new endpoint



method 2: table of values

$y=\sqrt{x} \rightarrow y=\sqrt{-x} \rightarrow y=\sqrt{-(x+4)} \rightarrow y=\sqrt{-(x+4)}-3$

x	y
0	0
1	1
4	2
9	3

→ negate x-values →

x	y
0	0
-1	1
-4	2
-9	3

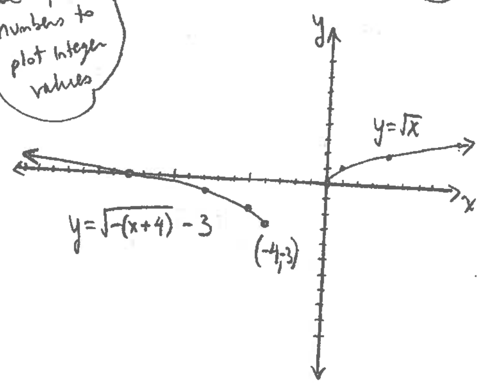
→ take away 4 from x-values →

x	y
-4	0
-5	1
-8	2
-13	3

→ take away 3 from y-values →

x	y
-4	-3
-5	-2
-8	-1
-13	0

make your life easy & use square numbers to plot integer values



method 3: mapping notation

Start from base fn $y=\sqrt{x}$ and map transformations to $y=\sqrt{-(x+4)}-3$

$(x,y) \rightarrow (-x,y)$ reflection in y-axis

$\rightarrow (-x-4,y)$ left 4

$\rightarrow (-x-4,y-3)$ down 3

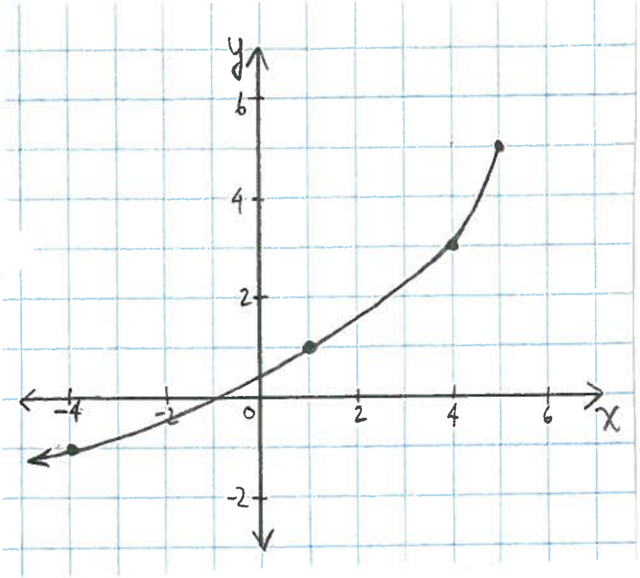
You may even "read" all transformations from $y=\sqrt{-(x+4)}-3$ in 1 step:

$$(x,y) \rightarrow (-x-4,\ y-3)$$

x	y	→	$-x-4$	$y-3$
0	0		$-(0)-4=-4$	$(0)-3=-3$
1	1		$-(1)-4=-5$	$(1)-3=-2$
4	2		$-(4)-4=-8$	$(2)-3=-1$
9	3		$-(9)-4=-13$	$(3)-3=0$

← Then plot points (see graph to left)

Ex 2 Determine a Radical Function from the graph below



tip: start backwards ☺

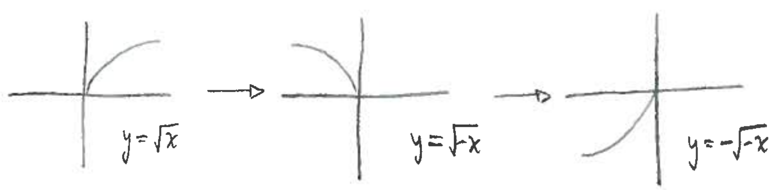
- What vertical & horizontal translations did it go through?
  → get c & d values (see endpoint!)
- Any reflections?
  → $a<0$ or $a>0$? (negative or positive)
  → $b<0$ or $b>0$? (negative or positive)
- then find $|a|$ OR $|b|$ value:
  → graphically -OR- algebraically

Sol'n: Recall standard form is $y=a\sqrt{b(x-c)}+d$ for a radical function

step 1: find c & d values by referring to endpoint

Endpoint is $(5,5)$, so $c=5$, $d=5$

step 2: determine any reflections (i.e., whether a & b values are positive or negative)



So $a<0$
$b<0$

step 3: By steps 1 & 2, we're looking for $y=-a\sqrt{-(x-5)}+5$ OR $y=-\sqrt{-b(x-5)}+5$

method 1

If we choose $y=-a\sqrt{-(x-5)}+5$ & a pt on graph; $(1,1)$

$$1=-a\sqrt{-(1-5)}+5$$
$$1=-a\sqrt{4}+5$$
$$-4=-2a$$
$$a=2$$

$\therefore y=-2\sqrt{-(x-5)}+5$

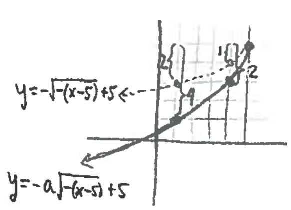
method 2

If we choose $y=-\sqrt{-b(x-5)}+5$ + a pt on graph; $(1,1)$

$$1=-\sqrt{-b(1-5)}+5$$
$$1=-\sqrt{4b}+5$$
$$-4=-2\sqrt{b}$$
$$\sqrt{b}=2$$
$$b=4$$

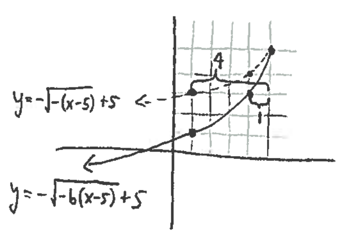
$\therefore y=-\sqrt{-4(x-5)}+5$

method 3 examining the graph vertically or horizontally



vertical distance × 2
1→2
2→4 ∴ $a=2$
$\therefore y=-2\sqrt{-(x-5)}+5$

-OR-



horizontal distance × $\frac{1}{4}$
4→1 ∴ $b=4$
$\therefore y=-\sqrt{-4(x-5)}+5$

HWK pg 72-75 # 3, 5, 6, 10, 16

2.2 Square Root of a function

Ex 1 Given $f(x) = 3-2x$, graph the functions $y=f(x)$ and $y=\sqrt{f(x)}$ on the same graph. Make a chart that compares the value of $f(x)$ & $\sqrt{f(x)}$ (note: these are y-values on the graph). Where do invariant points occur?

Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Value of $\sqrt{f(x)}$	$\sqrt{f(x)}$ is undefined	$\sqrt{f(x)} = 0$ Invariant points occur here	$\sqrt{f(x)} > f(x)$ Radical Fn ABOVE original Fn	$\sqrt{f(x)} = 1$ Invariant points occur here	$\sqrt{f(x)} < f(x)$ Radical Fn BELOW original Fn

* You can always just <u>take the square root</u> of original y-values ($y=f(x)$) to get corresponding y-values on $y=\sqrt{f(x)}$. Use a table of values. <u>Make sure to include x values where $f(x)=0$ + $f(x)=1$.</u>

must include:

i) $f(x) = 0$

$3-2x = 0$

$-2x = -3$

(1.5, 0) $x = 1.5$

ii) $f(x) = 1$

$3-2x = 1$

$-2x = -2$

(1, 1) $x = 1$

$y = 3-2x$

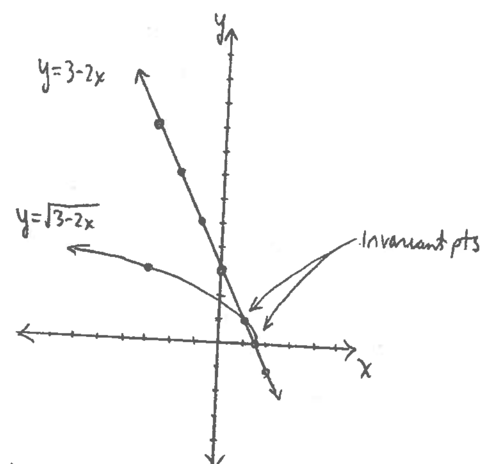
x	y
-3	$3-2(-3)=9$
-2	$3-2(-2)=7$
-1	5
0	3
1	1
1.5	0
2	-1

$\longrightarrow$

$y = \sqrt{3-2x}$

x	y
-3	$\sqrt{9} = 3$
-2	$\sqrt{7}$
-1	$\sqrt{5}$
0	$\sqrt{3}$
1	$\sqrt{1} = 1$
1.5	$\sqrt{0} = 0$
2	undefined

Can't take sqrt of negative number



Verify comparisons in chart with our graphs. Look for patterns to allow you to quickly sketch instead of making a table of values. Note invariant points at (1, 1) + (1.5, 0).

Ex2 Compare domain & range of the following functions

a) $y = 2 - 0.5x^2$ & $y = \sqrt{2 - 0.5x^2}$

b) $y = x^2 + 5$ & $y = \sqrt{x^2 + 5}$

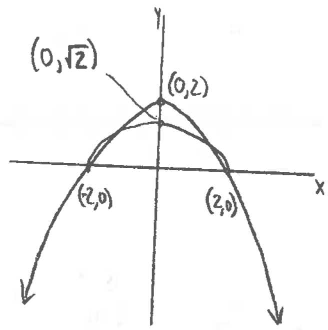
Sol'n: a) graphically

→ Use graphing calculator or graph manually (see Ex 3)

On graphing calc: (recommended)

Y1 = 2 − 0.5X^2

Y2 = √(2 − 0.5X^2)



$y = 2 - 0.5x^2$

D: $x \in \mathbb{R}$

R: $y \leq 2, y \in \mathbb{R}$ } easy to see from graph

$y = \sqrt{2 - 0.5x^2}$

D: $-2 \leq x \leq 2, x \in \mathbb{R}$ — x-int give restriction

R: $0 \leq y \leq \sqrt{2}, y \in \mathbb{R}$ — y-int may give min/max y-value

-OR-

analysing key points (algebraically)

x-intercepts & y-intercepts in $y = 2 - 0.5x^2$ will help determine D + R of $y = \sqrt{2 - 0.5x^2}$

x-int: set $y = 0$

$0 = 2 - 0.5x^2$

$0.5x^2 = 2$

$x^2 = 4$

$x = \pm 2$

∴ $(-2,0)$ + $(2,0)$ are x-int for both $y = 2 - 0.5x^2$ $y = \sqrt{2 - 0.5x^2}$

y-int: set $x = 0$, $y = 2$ — max y-value

∴ $\sqrt{2}$ max y-value for $y = \sqrt{2 - 0.5x^2}$

min value for $y = \sqrt{2 - 0.5x^2}$ is 0 ... why?

$y = 2 - 0.5x^2$

D: $x \in \mathbb{R}$

R: $y \leq 2, y \in \mathbb{R}$

$y = \sqrt{2 - 0.5x^2}$

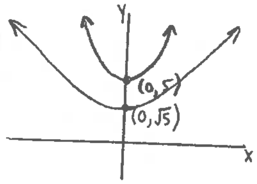
D: $-2 \leq x \leq 2, x \in \mathbb{R}$

R: $0 \leq y \leq \sqrt{2}, y \in \mathbb{R}$

b) graphically

set Y1 = X^2+5

Y2 = √(X^2+5)



$y = x^2 + 5$

D: $x \in \mathbb{R}$

R: $y \geq 5, y \in \mathbb{R}$

$y = \sqrt{x^2 + 5}$

D: $x \in \mathbb{R}$

R: $y \geq \sqrt{5}, y \in \mathbb{R}$

-OR-

analysing key points

	$y = x^2 + 5$	$y = \sqrt{x^2 + 5}$
x-int	none	none
y-int	5	$\sqrt{5}$
max	none	none
min	5	$\sqrt{5}$

x-int: $0 \neq x^2 + 5$ ∴ none for either fn ∴ no max/min

y-int: $y = 0^2 + 5$ so $y = 5$ for $y = x^2 + 5$ + $y = \sqrt{5}$ for $y = \sqrt{x^2 + 5}$

∴ min 5 + $\sqrt{5}$, respective no max

$y = x^2 + 5$

D: $x \in \mathbb{R}$

R: $y \geq 5, y \in \mathbb{R}$

$y = \sqrt{x^2 + 5}$

D: $x \in \mathbb{R}$

R: $y \geq \sqrt{5}, y \in \mathbb{R}$

Ex 3 Given $f(x) = x^2 - 3$, sketch $y = \sqrt{f(x)}$ from the graph of $y = f(x)$

Sol'n: Step 1: graph $f(x) = x^2 - 3$ and mark invariant points where $f(x) = 0$ + 1

i) $f(x) = 0 \Rightarrow 0 = x^2 - 3$

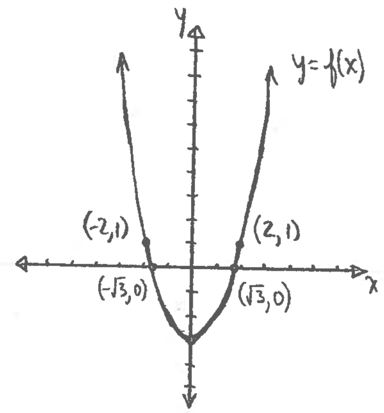
$\sqrt{3} = \sqrt{x^2}$

$\pm\sqrt{3} = x \quad \therefore (-\sqrt{3}, 0) + (\sqrt{3}, 0)$

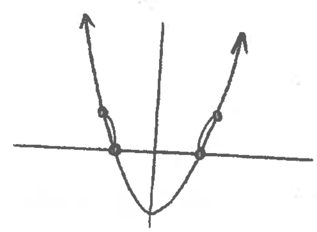
ii) $f(x) = 1 \Rightarrow 1 = x^2 - 3$

$\sqrt{4} = \sqrt{x^2}$

$\pm 2 = x \quad \therefore (-2, 1) + (2, 1)$

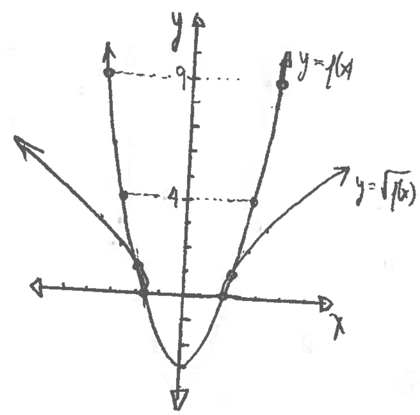


Step 2: draw the smooth curve of $y = \sqrt{f(x)}$ between invariant points ABOVE graph of $y = \sqrt{f(x)}$

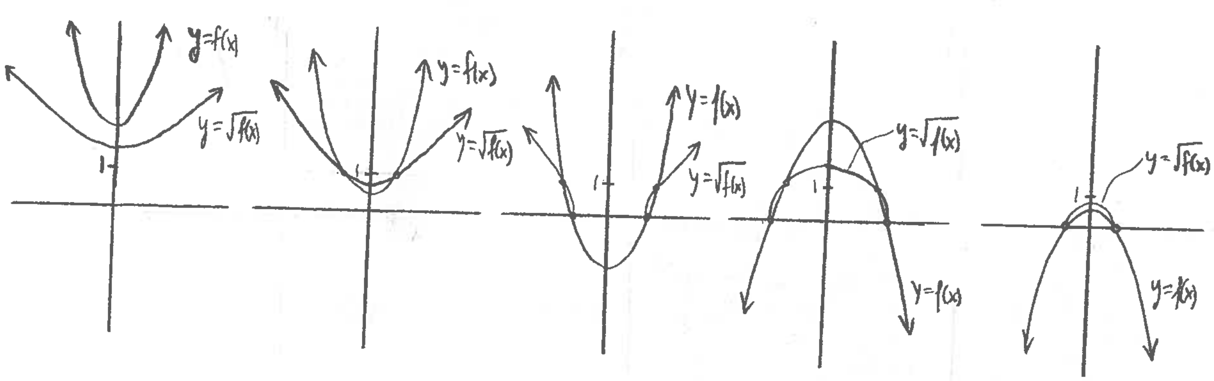


Step 3: Plot key points of $y = \sqrt{f(x)}$ by finding y-values of $y = f(x)$ that are square (then $y = \sqrt{f(x)}$ will be integer values)

Step 4: Connect points with smooth curve. Notice it looks almost linear — why?



There are 5 basic graph shapes based on whether the parabola opens up or down, and depending on the position of the vertex, and if there are any x-int.



If parabola, $y = f(x)$, is entirely below the x-axis, $y = \sqrt{f(x)}$ is undefined

Here, $f(x) = ax^2 + c$. What happens when the vertex of the parabola is on (0,1) or (0,0)? What is the relationship between the vertex point of the parabola and the max/min value of $y = \sqrt{f(x)}$?

$\boxed{\text{HWK: pg 87–89 \# 3, 5, 8, 11, 16}}$