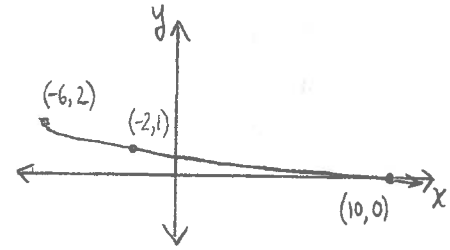
Day 5

Review: ① Write the equation of a radical function of the form $y = a\sqrt{b(x-c)} + d$ for the graph to the right.



* alternate form (more applied): "Determine the eqn of a radical fn with endpoint (-6,2) and an x-intercept of -6."
(no pic would be provided)

② If $(-4, 17)$ is a point on the graph of $y = f(x)$, identify a point on the graph of $y = -2\sqrt{f(x+3)-1} + 4$.

Soln: ① Either form can be done algebraically, but only the version with the graph can be done graphically.

Step 1: get c, d values by using endpoint $(-6, 2)$. Thus, $c = -6$, $d = 2$

Step 2: Any reflections? We can see $y = \sqrt{x}$ has been reflected through x-axis $\therefore$ $a < 0$

* In alternate form, since we don't have a graph, we will focus on whether $y = a\underbrace{\sqrt{b(x+6)}}_{\text{this}} + 2$ is defined for $b > 0$ or $b < 0$. Using $(10, 0)$, for example, we see this is positive (defined) only when $b > 0$.

Step 3: Let $b = 1$ ($\because b > 0$) and determine a value. (upon solving for a, we will see $a < 0$ like in graph)

Use $y = a\sqrt{(x+6)} + 2$ & point on graph. $(10, 0)$ is x-int & works fine.

$$0 = a\sqrt{10+6} + 2$$
$$0 = a\sqrt{16} + 2$$
$$-2 = 4a$$
$$a = -\tfrac{1}{2}$$

$\therefore$ $y = -\frac{1}{2}\sqrt{x+6} + 2$ satisfies the graph.

② Easiest to see in two parts: $y = f(x) \xrightarrow{(i)} y = f(x+3) - 1 \xrightarrow{(ii)} y = -2\underbrace{\sqrt{f(x+3)-1}} + 4$

sqrt of y-value; then apply transformations like $y = -2\sqrt{g(x)} + 4$, where $g(x) = f(x+3) - 1$

(i) $(x, y) \longrightarrow (x-3, y-1)$

(ii) $(x-3, y-1) \longrightarrow (x-3, -2\sqrt{y-1} + 4)$

overall: $(x, y) \longrightarrow (x-3, -2\sqrt{y-1} + 4)$

So $(-4, 17) \longrightarrow ((-4)-3, -2\sqrt{(17)-1} + 4) = (-7, -4)$ $\therefore$ $(-7, -4)$ is a pt on $y = -2\sqrt{f(x+3)-1} + 4$

2.3 Solving Radical Eqns graphically (& algebraically ☺)

Ex1 Determine root(s) of $\sqrt{x-4} = 5$

Soln First determine restrictions on x, if any (find domain).

$$x - 4 \geq 0$$
$$\therefore x \geq 4$$

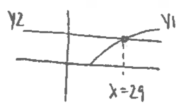
Then find roots...

I) algebraically

$$\sqrt{x-4} = 5$$
$$(\sqrt{x-4})^2 = 5^2$$
$$x - 4 = 25$$
$$x = 29$$

-OR-

II) graphically



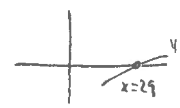
method 1

$$Y1 = \sqrt{(X-4)}$$
$$Y2 = 5$$

Then use $\boxed{\text{Intersect}}$ feature:

$\boxed{\text{2nd}}$ $\boxed{\text{Calc}}$ 5: intersect $x=29$

method 2



$$Y1 = \sqrt{(X-4)} - 5$$ (need to rearrange eqn)

Then use $\boxed{\text{Zero}}$ feature:

$\boxed{\text{2nd}}$ $\boxed{\text{Calc}}$ 2: zero (need to pick left + right bound on either side of zero)

$x = 29$

* You will need to adjust your WINDOW accordingly

Ex2 Solve the equation $\sqrt{x+5} = x + 3$

Soln $x + 5 \geq 0 \Rightarrow x \geq -5$ (restriction on x)

method 1 (algebraically)

$$\sqrt{x+5} = x + 3$$
$$(\sqrt{x+5})^2 = (x+3)^2$$
$$x + 5 = (x+3)(x+3)$$

	x	$+3$
x	x^2	$3x$
$+3$	$3x$	3^2

$$x + 5 = x^2 + 3x + 3x + 3^2$$
$$x + 5 = x^2 + 6x + 9$$
$$0 = x^2 + 5x + 4$$
$$0 = x^2 + 4x + x + 4$$
$$0 = x(x+4) + 1(x+4)$$
$$0 = (x+4)(x+1)$$
$$\therefore x + 4 = 0 \text{ or } x + 1 = 0$$
$$x = -4 \qquad x = -1$$

Check both roots!

✓ both in domain

Check $x = -4$:

$LS = \sqrt{-4+5}$ $\quad RS = -4 + 3$

$= \sqrt{1}$ $\quad = -1$

$= 1$

$\therefore LS \neq RS$ $\quad x \neq -4$ extraneous root!

Check $x = -1$:

$LS = \sqrt{-1+5}$ $\quad RS = -1 + 3$

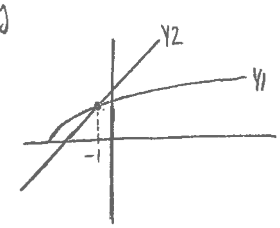
$= \sqrt{4}$ $\quad = 2$

$= 2$

$\because LS = RS$ $\quad x = -1$ ✓

method 2 graphically

$$Y1 = \sqrt{(X+5)}$$
$$Y2 = X + 3$$



Using $\boxed{\text{INTERSECT}}$

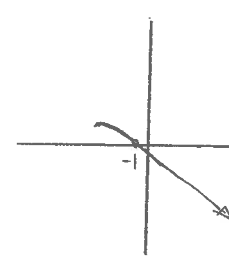
-OR-

$$Y1 = \sqrt{x+5} - x - 3$$

$\sqrt{x+5} = x + 3$

$\sqrt{x+5} - x - 3 = 0$

Using $\boxed{\text{ZERO}}$



NOTE: No extraneous roots when graphing!! ☺

Ex 3 Solve $\sqrt{3x^2-5} = x+4$ graphically. Verify solution algebraically.

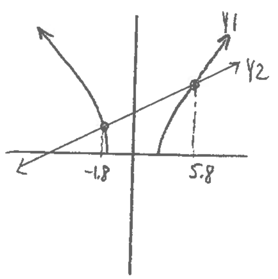
method 1:

$Y1 = \sqrt{(3X^2-5)}$

$Y2 = X+4$

Use $\boxed{\text{INTERSECT}}$ function

$X_1 \approx -1.8$ + $X_2 \approx 5.8$

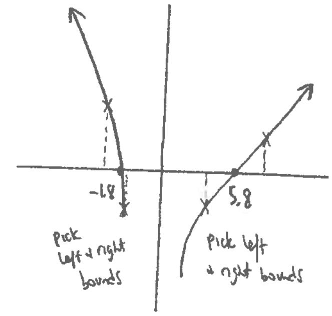


method 2:

$Y1 = \sqrt{(3X^2-5)} - X - 4$

Use $\boxed{\text{ZERO}}$ function

$X_1 \approx -1.8$ + $X_2 \approx 5.8$



Algebraically:

$$3x^2 - 5 \geq 0$$
$$3x^2 \geq 5$$
$$\sqrt{x^2} \geq \sqrt{\tfrac{5}{3}}$$
$$|x| \geq \sqrt{\tfrac{5}{3}}$$
$$\therefore\ x \leq -\sqrt{\tfrac{5}{3}} \text{ or } x \geq \sqrt{\tfrac{5}{3}}$$
$$x \leq -1.3 \qquad x \geq +1.3$$

$$\sqrt{3x^2-5} = x+4$$
$$\left(\sqrt{3x^2-5}\right)^2 = (x+4)^2$$
$$3x^2 - 5 = x^2 + 8x + 16$$
$$2x^2 - 8x - 21 = 0$$

Doesn't factor nicely
— use quadratic formula!

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 2$
$b = -8$
$c = -21$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-21)}}{2(2)}$$
$$= \frac{8 \pm \sqrt{232}}{4}$$
$$= \frac{8 \pm 2\sqrt{58}}{4}$$
$$= \frac{4 \pm \sqrt{58}}{2}$$

$\sqrt{232} = \sqrt{4 \times 58}$
$= \sqrt{4}\sqrt{58}$
$= 2\sqrt{58}$

So $x = \frac{4-\sqrt{58}}{2}$ or $x = \frac{4+\sqrt{58}}{2}$

$x \approx -1.8$ ✓ $\qquad x \approx 5.8$ ✓

* check restriction
** plug back into eqn to see if any is extraneous

Note: * graphical method faster but often approximate
** algebraic method often longer but exact (though there may be extraneous roots)

HWR: pg 96-98 # 5-7, 15, 16