Day 6 Review

① Show $6+\sqrt{x+4}=2$ has no solution

② Solve $45-\sqrt{10+2x^2}=25$. Find an exact solution (i.e. algebraically). Verify graphically.

③ (p98 #11) $y=\sqrt{-3(x+c)}+c$ passes through $(-1,1)$. Solve for c.

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① $6+\sqrt{x+4}=2$ ⟶ OR we know $\sqrt{x+4}\geq 0$, so $6+\sqrt{x+4}\geq 6$

Thus $6+\sqrt{x+4}\neq 2$

$\sqrt{x+4}=-4$

$\left.\begin{aligned}(\sqrt{x+4})^2&=(-4)^2\\ x+4&=16\\ x&=12\end{aligned}\right\}$ false logic b/c $\sqrt{x+4}\neq -4$ (from line #2)

Indeed, when we plug in $x=12$: $LS=6+\sqrt{12+4}=6+\sqrt{16}=10$ + $RS=2$ ∴ $LS\neq RS$, no solution!

② $45-\sqrt{10+2x^2}=25$

$(20)^2=(\sqrt{10+2x^2})^2$

$400=10+2x^2$

$390=2x^2$

$\pm\sqrt{195}=\sqrt{x^2}$

$x=\pm\sqrt{195}$ ← exact!

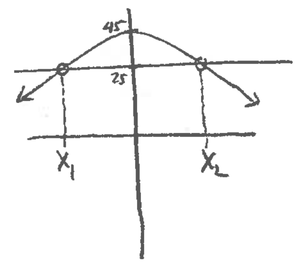
verify $Y1=45-\sqrt{(10+2X\wedge 2)}$

$Y2=25$

use $\boxed{\text{INTERSECT}}$

$X_1\approx -13.96$

$X_2\approx 13.96$ ← approximate



③ sub $(-1,1)$ i.e., $x=-1$, $y=1$ into eqn:

$(1)=\sqrt{-3[(-1)+c]}+c$

$1-c=\sqrt{-3(-1+c)}$

$(1-c)^2=-3(-1+c)$

$1-2c+c^2=3-3c$

$\underline{-3+3c\qquad -3+3c}$

$-2+c+c^2=0$ ✩

$(c+2)(c-1)=0$

∴ $c+2=0$ or $c-1=0$

$\boxed{c=-2}$ $\boxed{c=1}$

aside: $(1-c)^2=(1-c)(1-c)$

use area model:

	1	$-c$
1	1	$-c$
$-c$	$-c$	c^2

$1-c-c+c^2$

$=1-2c+c^2$

$-3(-1+c)$

	-1	c
-3	3	$-3c$

$3-3c$

"trick": use FOIL

multiply terms in this order

First
Outer
Inner
Last

$(1-c)(1-c)$

✩ $-2+c+c^2=0$

$c^2+c-2=0$

$c^2+2c-1c-2=0$

$c(c+2)-1(c+2)=0$

$(c+2)(c-1)=0$

Unit 1 (ch1+2) Review: ch1 p56/57 #1-17 ; ch 2 p99-101 #2-17

Ch 3: Polynomial FNS

3.1 Characteristics of Polynomial FNS

The textbook has an excellent summary of this section on pg 109, so why re-invent the wheel...

Key Ideas

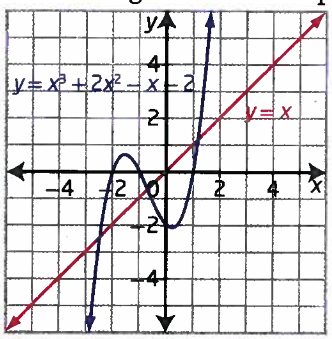
- A polynomial function has the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$, where a_n is the leading coefficient; a_0 is the constant; and the degree of the polynomial, n, is the exponent of the greatest power of the variable, x.

$a_n \in \mathbb{R}$

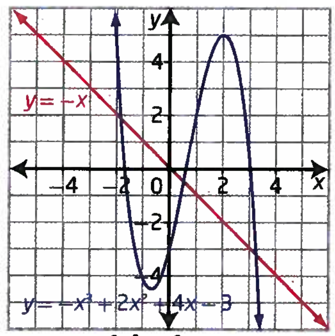
$n \in \mathbb{N}$

(ie, $n = 0, 1, 2, \ldots$)

- Graphs of odd-degree polynomial functions have the following characteristics:
  - a graph that extends down into quadrant III and up into quadrant I (similar to the graph of $y = x$) when the leading coefficient is positive



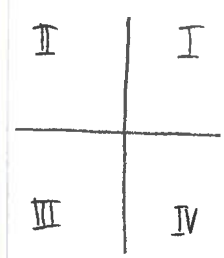
  - a graph that extends up into quadrant II and down into quadrant IV (similar to the graph of $y = -x$) when the leading coefficient is negative



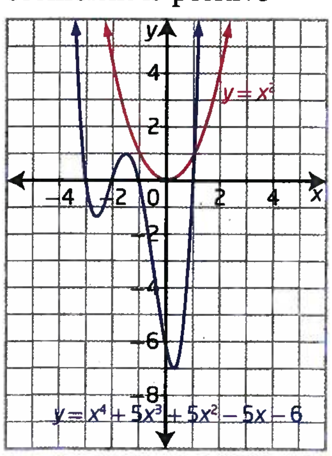
leading term n coefficient is the one of highest degree

  - a y-intercept that corresponds to the constant term of the function
  - at least one x-intercept and up to a maximum of n x-intercepts, where n is the degree of the function
  - a domain of $\{x \mid x \in R\}$ and a range of $\{y \mid y \in R\}$
  - no maximum or minimum points

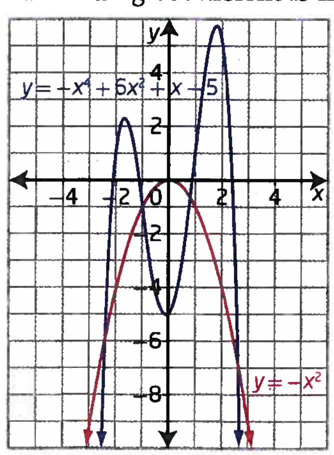
Quadrants:



- Graphs of even-degree polynomial functions have the following characteristics:
  - a graph that extends up into quadrant II and up into quadrant I (similar to the graph of $y = x^2$) when the leading coefficient is positive



  - a graph that extends down into quadrant III and down into quadrant IV (similar to the graph of $y = -x^2$) when the leading coefficient is negative



  - a y-intercept that corresponds to the constant term of the function
  - from zero to a maximum of n x-intercepts, where n is the degree of the function
  - a domain of $\{x \mid x \in R\}$ and a range that depends on the maximum or minimum value of the function

Ex 1 Polynomial FN or not? Why?

a) $g(x) = \sqrt{x} - 2$

b) $f(x) = -2x^3$

c) $y = |x|$

d) $y = 5x^3 - 2x - 1$

e) $y = \dfrac{3x^2 - x + 5}{2}$

f) $h(x) = \dfrac{x-1}{x}$

g) $y = \sqrt{2}x$

h) $k(x) = 0$

Soln:

a) No. $\sqrt{x} = x^{\frac{1}{2}}$ — not natural number

b) Yes. Degree 3. Leading term -2. Constant term 0

c) No. Cannot be written in required form (unless piecewise)

d) Yes. Degree 3. Leading term 5. constant term -1.

e) Yes. Degree 2. Leading term $\frac{3}{2}$. Constant term $\frac{5}{2}$.

f) No. $h(x) = \frac{x-1}{x} = \frac{x}{x} - \frac{1}{x} = 1 - x^{-1}$ — not natural number

g) Yes. Degree 1. leading term $\sqrt{2} \in \mathbb{R}$. Constant 0.

h) Yes. Degree 0. Constant 0

Ex 2 Identify characteristics of each polynomial FN:

- ☐ degree of FN (state whether even/odd)
- ☐ end behaviour of graph (i.e. what quadrant the extreme ends of the graph are in)
- ☐ number of possible x-int
- ☐ max or min value possible?
- ☐ determine y-int

Then match with possible graph:

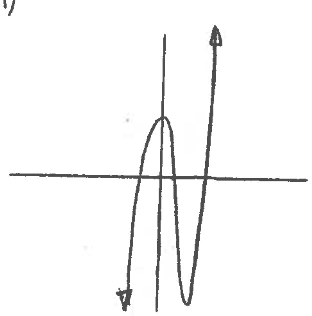
a) $f(x) = x^4 - x^3 + 4x^2 - 16x$

b) $g(x) = -4x^3 + 5x^2 - 3x + 5$

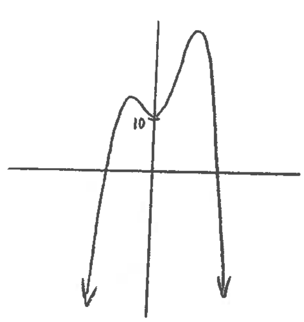
c) $m(x) = -x^4 + 5x^2 - 3x + 10$

d) $n(x) = 2x^5 - 4x^4 - 3x^2 - x + 5$

i)



ii)



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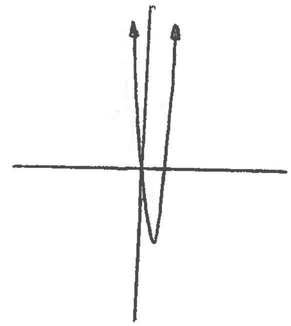

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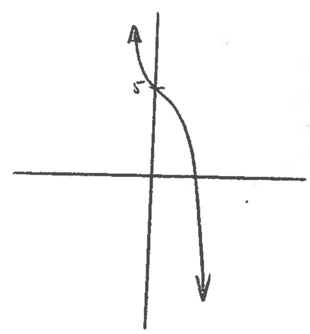
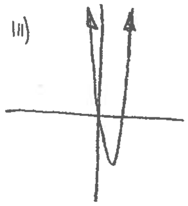


Soln: a) $f(x) = x^{4} - x^3 + 4x^2 - 16x$

iii)



(like $y = x^2$)

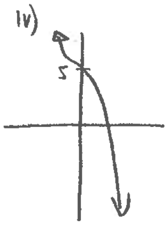
degree is 4 & positive leading term $\Longrightarrow$ quadrant $\underline{\text{II to I}}$ (end behaviour) + $\underline{\text{min}}$ value

↳ even degree means as little as 0 x-intercepts and up to 4 (b/c deg $\underline{4}$)

set $x = 0 \Longrightarrow f(0) = 0$ ie. y-int at $y = 0$.

b) $g(x) = -4x^{3} + 5x^2 - 3x + 5$

iv)



(like $y = -x^3$)

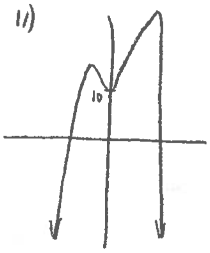
degree is 3 & negative leading term $\Longrightarrow$ quadrant $\underline{\text{II to I}}$ (end behaviour) + no min/max.

Odd degree means at least 1 x-intercept & up to 3 (b/c deg 3)

set $x = 0 \Longrightarrow g(0) = 5$ i.e. y-int is 5.

c) $m(x) = -x^4 + 5x^2 - 3x + 10$

ii)



(like $y = -x^2$)

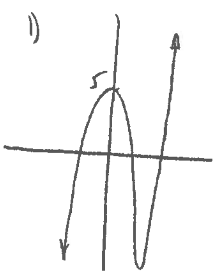
Deg 4 + neg. leading term $\Longrightarrow$ quad. III to IV (end behaviour) + $\underline{\text{max}}$ value

Deg 4 means 0 – 4 x-int possible.

y-int at 10 (why?)

d) $n(x) = 2x^5 - 4x^4 - 3x^2 - x + 5$

i)



(like $y = x^3$)

Deg 5 + pos. leading term $\Longrightarrow$ quad III to I for end behaviour + no min/max

Deg 5 $\Longrightarrow$ 1 to 5 x-int possible.

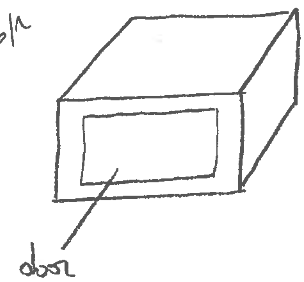
y-int is 5.

Ex 3 A toaster oven is built in the shape of a rectangular prism. Its volume, V, in cubic inches, is related to the height, h, in inches, of the oven door by the fn $V(h) = h^3 + 10h^2 + 31h + 30$.

a) What is the volume, in cubic inches, of the toaster oven if the oven door height is 8 in.?

b) What is the height of the oven door for the least toaster oven volume? Explain.

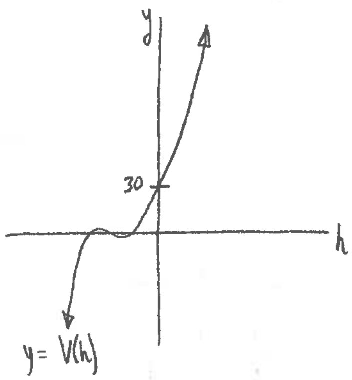
Sol'n



a) use formula & let $h = 8$ in.

$$V(8) = (8)^3 + 10(8)^2 + 31(8) + 30$$
$$= 1430 \text{ in}^3$$

Volume of door is 1430 in^3



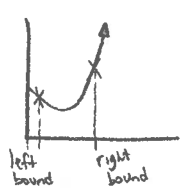
b) If you use your graphing calculator, you get the graph on the left.

However, context is important:

We can assume $h \geq 0$ & $V(h) \geq 0$ (negative values don't make sense)

So then the y-int, $y = 30$, is the minimum volume ... but then there would be no door at a height of 0 in.!

Note: for a graph like this...



You can find the local min (or relative min) using:

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ (CALC) 3: minimum

and choosing a left & right bound

HWK: pg 114 - 116 # 1-7, 9