Day 7 ☐ Review

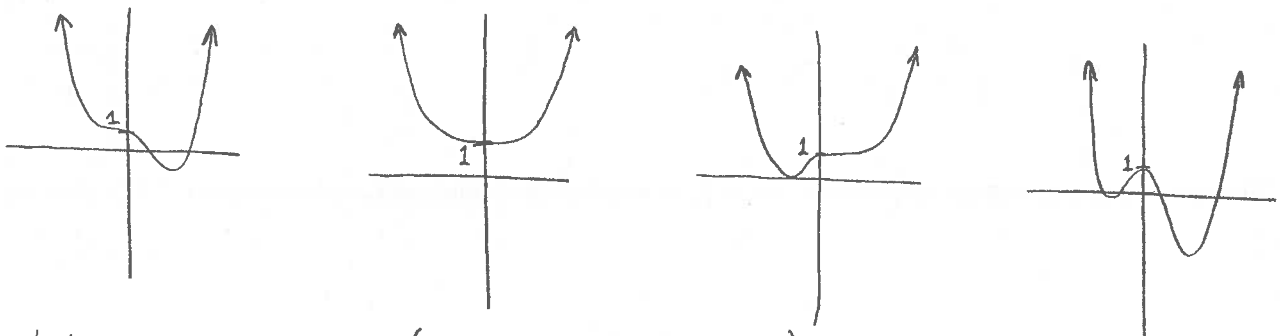
① What *might* the graph of this FN look like: $f(x) = 1 - x^3 + 2x^4 - 4x^2$?

② link to next section — Graph $V(x) = 2x^3 - 8x^2 + 6x$ with your graphing calculator. Then fully factor $V(x)$. How does the factored form relate to the graph?

Sol^n: ① Do not be fooled! Degree comes from highest power: $f(x) = 1 - x^3 + 2x^4 - 4x^2$ (y-int: 1)

So degree is 4 and leading coefficient is ⊕ with y-int of 1. End behaviour: quad II to I

The function can also have 0 to 4 roots. Here are some *possible* graphs:



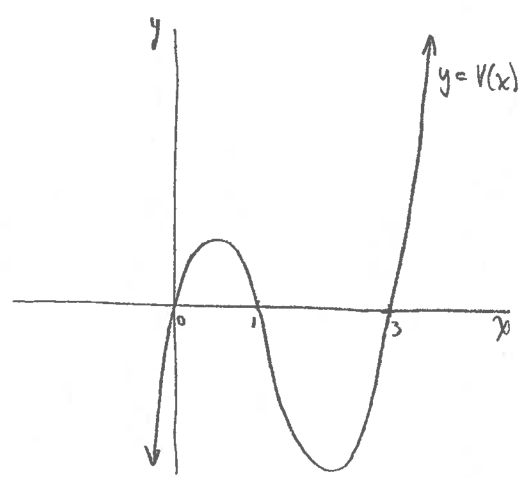
* The last graph is most accurate (can check with graphing calc.)

② $V(x) = 2x^3 - 8x^2 + 6x$

$V(x) = 2x(x^2 - 4x + 3)$

$V(x) = 2x(x-1)(x-3)$

roots: $x=0$, $x=1$, $x=3$



Notice that the factored form reveals the roots of $V(x)$, allowing us to see where $V(x)$ crosses the x-axis.

3.2 - Remainder Theorem

Recall long division ...

$$\begin{array}{r} 4 \\ 8\overline{)343} \\ -32 \\ \hline 23 \end{array} \quad \rightarrow \quad \begin{array}{r} 42 \\ 8\overline{)343} \\ -32 \\ \hline 23 \\ -16 \\ \hline 7 \end{array}$$

8×4 $\quad$ 8×2

terminology:

$$\begin{array}{r} 42 \\ 8\overline{)343} \\ -32 \\ \hline 23 \\ -16 \\ \hline 7 \end{array}$$

quotient, divisor, divided, remainder

Note:

- $8 \times 42 + 7 = 343$ (good as a "check")
- $\frac{343}{8} = 42 + \frac{7}{8}$
  $= 42\frac{7}{8}$ (standard form for result)

Now try with a polynomial divided by a binomial ...

$$\begin{array}{r} x \\ x+2\overline{)x^2+5x+13} \\ -(x^2+2x) \\ \hline 3x+13 \end{array} \quad \rightarrow \quad \begin{array}{r} x+3 \\ x+2\overline{)x^2+5x+13} \\ -(x^2+2x) \\ \hline 3x+13 \\ -(3x+6) \\ \hline 7 \end{array}$$

$x(x+2)$ $\quad$ $3(x+2)$

Note:

check: $(x+2)(x+3) + 7 = x^2+5x+13$

divisor $\quad$ quotient $\quad$ remainder $\quad$ divided

standard form: $\frac{x^2+5x+13}{x+2} = (x+3) + \frac{7}{x+2}$, $x \neq -2$ (why?)

In general...

$$\boxed{\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}}$$

$$\boxed{\text{check with: } (x-a)\,Q(x) + R = P(x)}$$

$P(x)$ is polynomial (divided)
$Q(x)$ is quotient
R is remainder
$x-a$ is divisor, $a \in \mathbb{Z}$

set of integers: $0, \pm1, \pm2, \ldots$

Ex 1 a) Divide $P(x) = 5x^3 + 10x - 13x^2 - 9$ by $x-2$.
Express as $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$

b) Identify any restrictions on x.

c) Verify your solution.

Soln:
a)

$$\begin{array}{r} 5x^2-3x+4 \\ x-2\overline{)5x^3-13x^2+10x-9} \\ 5x^3-10x^2 \\ \hline -3x^2+10x-9 \\ -3x^2+6x \\ \hline 4x-9 \\ 4x-8 \\ \hline -1 \end{array}$$

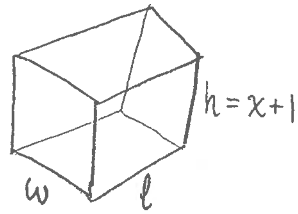
← not $P(x)$ put in order of descending powers

$\therefore \frac{5x^3+10x-13x^2-9}{x-2} = (5x^2-3x+4) + \left(\frac{-1}{x-2}\right)$

b) note $x-2 \neq 0$ $\therefore x \neq 2$
(can't divide by 0)

c) $(x-2)(5x^2-11x-12) + (-33)$
$= 5x^3 - 11x^2 - 12x - 2x^2 + 22x + 24 - 33$
$= 5x^3 - 13x^2 + 10x - 9$ ✓

Ex 2



Volume of box is $V(x) = x^3 + 7x^2 + 14x + 8$ where height (h) is $x+1$. Find possible dimensions of box.

Sol'n: we know $V(x) = l \times w \times h \implies \dfrac{V(x)}{h} = l \times w$ area of base

$$\begin{array}{r|l} & x^2 + 6x + 8 \\ \hline x+1 & x^3 + 7x^2 + 14x + 8 \\ & x^3 + x^2 \\ \hline & \quad 6x^2 + 14x + 8 \\ & \quad 6x^2 + 6x \\ \hline & \qquad\quad 8x + 8 \\ & \qquad\quad 8x + 8 \\ \hline & \qquad\qquad 0 \end{array}$$

no remainder! → 0

$$\implies V(x) = \underbrace{(x+1)}_{h}\underbrace{(x^2+6x+8)}_{l \times w}$$ factor ☺

$$V(x) = (x+1)(x+2)(x+4)$$
$$\quad h \times l \times w$$

∴ Possible dimensions are $x+1$, $x+2$, $x+4$.

Synthetic Division

This is a time saving device that is an alternative to long division.

Ex 3. $2x^4 + 5x^3 - 1 + 4x$ divided by $x+3$

Step 1: Put in order of descending powers. Use 0 for any missing powers. Then write in coefficients and factor.

$2x^4 + 5x^3 + 0x^2 + 4x - 1x$

$$\begin{array}{c|ccccc} +3 & 2 & 5 & 0 & 4 & -1 \\ - & & & & & \\ \hline \times & & & & & \end{array}$$

from factor $x+3$

to indicate we'll subtract

to indicate multiplying by +3

Step 2: Bring down first coefficient (ex 2 here), then multiply by constant in factor (ex +3 here). Subtract result from 2nd coefficient (ex 0 here). Continue in this way until remainder reached.

$$\begin{array}{c|ccccc} +3 & 2 & 5 & 0 & 4 & -1 \\ - & \downarrow & 6 & & & \\ \hline \times & 2 & -1 & & & \end{array}$$

$(3)(2) = 6$ $\qquad 5-6=-1$

continue →

$$\begin{array}{c|ccccc} +3 & 2 & 5 & 0 & 4 & -1 \\ - & & 6 & -3 & 9 & -15 \\ \hline \times & 2 & -1 & 3 & -5 & 14 \end{array}$$

remainder

Step 3: Give solution ...

note powers begin at 1 less than degree of P(x)

$$\frac{2x^4 + 5x^3 - 1 + 4x}{x+3} = 2x^3 - x^2 + 3x - 5 + \frac{14}{x+3}$$

* Try $\dfrac{2x^3 + 3x^2 - 4x + 15}{x+3}$ using synthetic division (full solution pg 122 in text)

Remainder Theorem:

When a polynomial in x, $P(x)$, is divided by $x-a$, the remainder is $P(a)$.

Ex 4. a) Use remainder theorem to find the remainder when $P(x) = x^3 - 10x + 6$ is divided by $x+4$

b) Verify solution using synthetic division.

Sol^{n}: $x + 4 = x - (-4) \Rightarrow a = -4$

So then $P(-4) = (-4)^3 - 10(-4) + 6$

$= -64 + 40 + 6$

$= -18$

$\therefore$ remainder is -18.

b) Rewrite $P(x) = x^3 - 10x + 6$ as $P(x) = 1x^3 + 0x^2 - 10x + 6$. Divided by $x+4$

$$
\begin{array}{c|cccc}
+4 & 1 & 0 & -10 & 6 \\
- & & 4 & -16 & 24 \\
\hline
\times & 1 & -4 & 6 & -18
\end{array}
$$

remainder verified

HWR: p124 #1, 2, 5, 7, 9–12, 14, 16