Day 9 Review

① A rectangular prism has volume: $V(x) = x^3 + 7x^2 - 28x + 20$, where $V(x)$ is in ft^3 and x is a positive real number. What are the possible dimensions (in x) of this prism?

② pg 135 #15: Determine m & n so that $2x^3 + mx^2 + nx - 3$ & $x^3 - 3mx^2 + 2nx + 4$ are both divisible by $x-2$.

Sol'n ① $V(x) = x^3 + 7x^2 - 28x + 20$

possible zeros $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$V(1) = (1)^3 + 7(1)^2 - 28(1) + 20$
$= 0$

$\therefore x-1$ is a factor

$$\begin{array}{c|cccc} -1 & 1 & 7 & -28 & 20 \\ - & & -1 & -8 & 20 \\ \hline \times & 1 & 8 & -20 & 0 \end{array}$$

should be 0!

$\therefore x^2 + 8x - 20$ is a factor

So then $V(x) = (x-1)(x^2+8x-20)$
$V(x) = (x-1)(x-2)(x+10)$

$\therefore$ possible demensions are $x-1,\ x-2,\ x+10$ ft.

OR graph $Y1 = X\wedge3 + 7X\wedge2 - 28X + 20$ to find roots: $1, 2, -10$ + get same factors ☺

② $x-2$ divides into each means $a=2$ where:

$2(2)^3 + m(2)^2 + n(2) - 3 = 0 \longrightarrow 4m + 2n + 13 = 0$ ①

$(2)^3 - 3m(2)^2 + 2n(2) + 4 = 0 \longrightarrow -12m + 4n + 12 = 0 \xrightarrow{\div 2} -6m + 2n + 6 = 0$ ②

2 eqns + 2 unknowns — System of equations! Do ① − ②:

$$\begin{array}{r} 4m + 2n + 13 = 0 \\ -(-6m + 2n + 6 = 0) \\ \hline 10m \qquad + 7 = 0 \end{array} \Longrightarrow \boxed{m = \frac{-7}{10}} \ ③$$

Sub ③ into ②: $-6\left(\frac{-7}{10}\right) + 2n + 6 = 0$

$\frac{42}{10} + 2n + \frac{60}{10} = 0$

$2n = \frac{-102}{10}$

$\boxed{n = \frac{-51}{10}}$

—OR— LONG WAY:

$$\begin{array}{c|cccc} -2 & 2 & m & n & -3 \\ & & -4 & -2m-8 & -2n-4m-16 \\ \hline & 2 & m+4 & n+2m+8 & -3+2n+4m+16 \end{array}$$

$\therefore 2n + 4m + 13 = 0$

$$\begin{array}{c|cccc} -2 & 1 & -3m & 2n & 4 \\ & & -2 & 6m-4 & -4n+12m-8 \\ \hline & 1 & -3m+2 & 2n-6m+4 & 4+4n-12m+8 \end{array}$$

$\therefore 4n - 12m + 12 = 0$

* then solve system of eqns like above

*** **NB:** You MUST do it this way if the remainder is nonzero!
i.e. use synthetic or long division

3.4 Equations + Graphs of Polynomial FNS

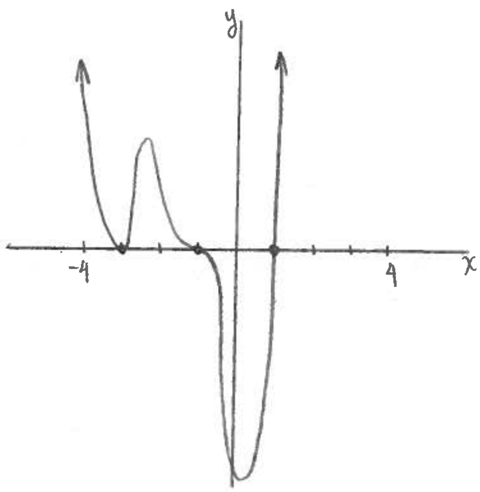
If $P(x)$ has a factor $(x-a)$ that is repeated n times, then we say $x=a$ is a zero (or root) of multiplicity n.

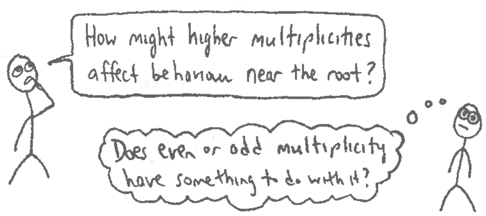
Ex $P(x) = (x-1)^{1}(x+1)^{3}(x+3)^{2}$

$x=1$ is a zero of multiplicity 1

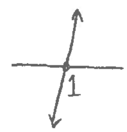
$x=-1$ is a zero of multiplicity 3

$x=-3$ is a zero of multiplicity 2

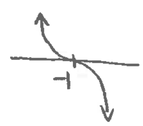




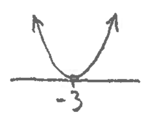
Notice the behavior at each zero.
How is it related to its multiplicity?



$x=1$ $\quad (x-1)^{1}$ — multiplicity 1 → linear near the zero (think $y=x^1$)



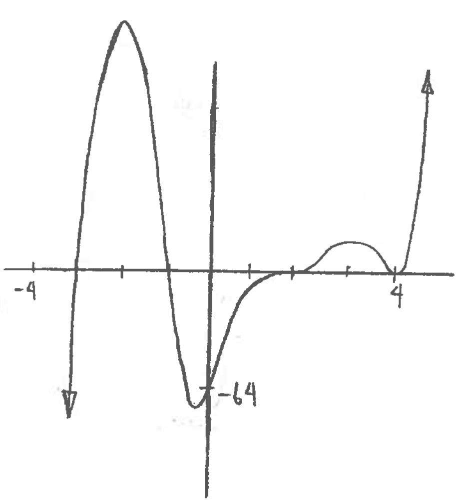
$x=-1$ $\quad (x+1)^{3}$ — multiplicity 3 → Inflection at zero (think $y=x^3$)

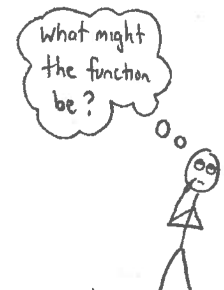


$x=-3$ $\quad (x+3)^{2}$ — multiplicity 2 → quadratic near the zero (think $y=x^2$)

Ex 1: Given the graph below, find:

- the least possible degree
- the sign of the leading coefficient
- x-intercept + factors of FN with least possible degree
- intervals where FN is positive + intervals where FN is negative.





Soln:

x-int	-3	-1	2	4
factors	$x+3$	$x+1$	$x-2$	$x-4$
multiplicity	1	1	3	2

least possible degree: $1+1+3+2=7$ (add up multiplicities)

Sign of leading coeff.: positive ∵ quad III to quad I

FN positive on: $-3<x<-1$, $2<x<4$, $x>4$

FN negative on: $x<-3$, $-1<x<2$

Ex 2 Sketch the graph of each polynomial FN:

a) $y = -(x+1)^3(x-3)$

b) $y = 2x^4 - 6x^2 + 4x$

Sol^n a) Degree: 4 (add up multiplicities: $3+1=4$) & negative leading coeff. $\Rightarrow$ quad III to quad IV

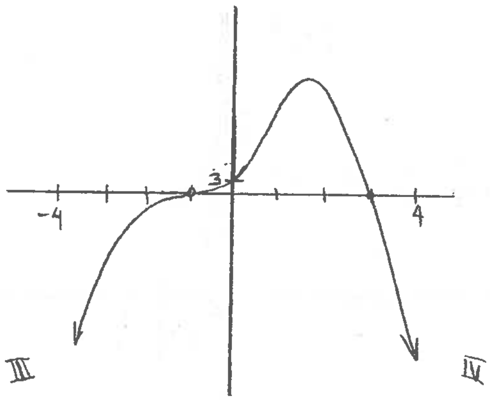
Zeros: $x=-1$, $x=3$

y-int: $-(0+1)^3(0-3) = -(1)^3(-3) = 3$

behaviour at zeros:

$(x+1)^3 \Rightarrow$ inflection @ $x=-1$

$(x-3)^1 \Rightarrow$ linear near $x=3$



b) factor first: $y = 2x^4 - 6x^2 + 4x$

$y = 2x(x^3 - 3x + 2)$

let $P(x) = x^3 - 3x + 2$ — possible values of a: $\pm 1, \pm 2$

try $a=1$ $P(1) = (1)^3 - 3(1) + 2$
$= 0$

$\therefore (x-1)$ is a factor of $P(x)$

So divide: $P(x) \div (x-1)$

$$\begin{array}{c|cccc} -1 & 1 & 0 & -3 & 2 \\ - & & -1 & -1 & 2 \\ \hline x & 1 & 1 & -2 & 0 \end{array}$$

better get 0 !!

$\therefore x^2 + x - 2$ is another factor

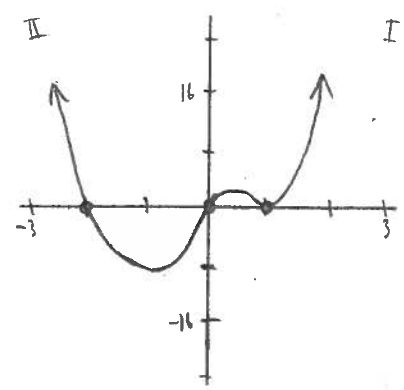
That is, $y = 2x(x-1)(x^2+x-2)$

$y = 2x(x-1)[(x-1)(x+2)]$

$y = 2x^1(x-1)^2(x+2)^1$

linear quadratic linear

After analysis:



Analysis:

Degree is $1+2+1=4$ (add up multiplicities)
positive leading coefficient
$\}\therefore$ quad II to I

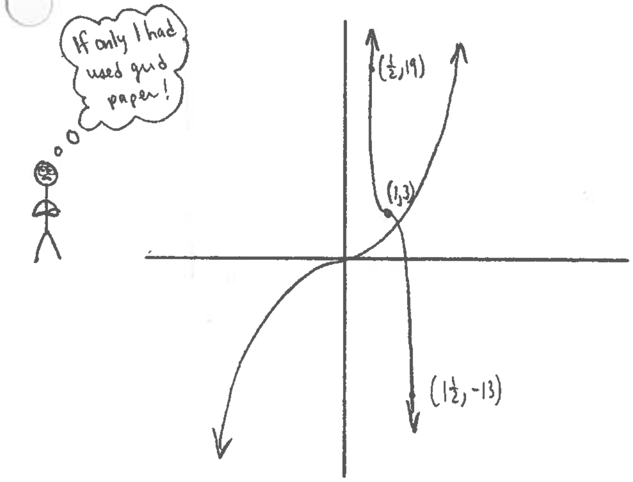
y-int: $2(0)(0-1)^2(0+2) = 0$

Zeros: $0, +1, -2$
mult: $1 \quad 2 \quad 1$
} tell us about behaviour at roots

Ex 3 Sketch a graph of $y = -2(4(x-1))^3 + 3$

Soln Review! Use mapping notation: $(x, y) \longrightarrow \left(\frac{x}{4} + 1, -2y + 3\right)$

$\left(y = x^3 \longrightarrow y = -2(4(x-1))^3 + 3 \right)$ — look up in Ch 1 if you forgot ☹



x	y
-2	-8
-1	-1
0	0
1	1
2	8

$\longrightarrow$

x	y	
$\frac{(-2)}{4} + 1 = \frac{1}{2}$	$-2(-8) + 3 = 19$	$\longrightarrow (\frac{1}{2}, 19)$
$\frac{(-1)}{4} + 1 = \frac{3}{4}$	$-2(-1) + 3 = 5$	$\longrightarrow (\frac{3}{4}, 5)$
$\frac{0}{4} + 1 = 1$	$-2(0) + 3 = 3$	$\longrightarrow (1, 3)$
$\frac{1}{4} + 1 = 1\frac{1}{4}$	$-2(1) + 3 = 1$	$\longrightarrow (1\frac{1}{4}, 1)$
$\frac{2}{4} + 1 = 1\frac{1}{2}$	$-2(8) + 3 = -13$	$\longrightarrow (1\frac{1}{2}, -13)$

Ex 4 Three consecutive integers have a product of -210.

a) Write a polynomial function to model this situation

b) What are the three integers?

Soln:

a) let x, $x+1$, $x+2$ be the consecutive integers.

Then ... $x(x+1)(x+2) = -210$

b) Make your life easier — graph it with your calculator!

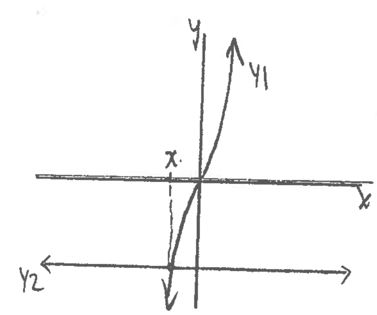
$Y1 = X(X+1)(X+2)$

$Y2 = -210$

Use $\boxed{\text{INTERSECT}}$ Function

Possible window

	min	max
X	-50	50
Y	-250	250



$x = -7$

$\therefore$ three integers are $-7, -6, -5$

HWK: pg 147 - 151 # 4, 5, 6c, 7, 8, 12, 14, 16