Pre-Calculus 12 Final Exam Review

TRANSFORMATIONS

1. Describe the transformations, in an appropriate order, that would take you from the graph of $y=f(x)$ to the graph of $y=-2f(2-4x)+3$.
2. Determine the inverse of the function $f(x)=\frac{2+x}{3x}$. Is the inverse also a function?
3. (3, –2) is a point on the graph of $y=f(x)$. Determine a point on the graph of $y=-f(-2x)+1$.
4. Suppose $f(x)=|x|$. Graph $y=f(x)$ and $y=-0.5f(x+2)+2$ on the same set of axes.
5. Determine the equation of the function $y=-|x|+3$ that is reflected through the line:
   a) $x=0$ b) $y=-1$ c) $x=2$

RADICAL & POLYNOMIAL FUNCTIONS

1. Suppose $f(x)=-3(2x+6)^2+12$. Graph $y=f(x)$ and $y=\sqrt{f(x)}$ on the same set of axes.
2. Consider the point (x, y) on the graph of the function $y=f(x)$. Use mapping notation to show its image point under the transformation described by $y=-\sqrt{-f(x+2)}+1$.
3. Factor $h(x)=-x^3+x^2+5x+3$ as much as possible. Explain how you could do this both algebraically and graphically.
4. Solve the radical equation: $\frac{1}{2}(x+25)=\sqrt{625-x^2}$ graphically and algebraically.
5. Provide a possible equation for an odd-degree polynomial function with a y-intercept of –2 and three unique roots of different multiplicities.

RATIONAL & COMPOSITE FUNCTIONS

1. Consider the functions $f(x)=x^2-x$ and $g(x)=x^2-3x+2$.
   a) Determine $(g\circ f)(-1)$.
   b) Write a simplified equation for $y=\left(\frac{f}{g}\right)(x)$. State any asymptotes, intercepts, and points of discontinuity.
   c) Graph $y=(f-g)(x)$. Describe two ways you could do this.
2. Sketch a graph of the rational function $y=\frac{x^2-x}{x^2-3x+2}$ without the use of technology.
3. Solve the following equation algebraically and graphically: $\frac{3x}{x+2}=x-\frac{6}{x+2}$. Then compare the solutions found using each method.
4. Consider the functions $f(x)=\frac{x}{x-1}$ and $g(x)=\frac{1}{2x}$. Write a simplified equation for $y=f(g(x))$. Then state the domain and range of this composite function.
5. You want a glass of 2% milk but only 1% milk and 3.25% milk are in the fridge. Cleverly, you dilute the milk fat in 200 ml of 3.25% milk by adding 1% milk. How much 1% milk do you need to add? What equation could you use to calculate the percent of milk fat, $M(x)$, when adding x ml of 1% milk to 200 ml of 3.25% milk? (Note that 2% milk means 2% milk fat (M.F.) by volume.)

TRIGONOMETRY

1. Determine a simplified expression for $\cos(\pi - x)$.
2. Develop the equation of a sinusoidal function that has a minimum of 0 and two consecutive crests at (3,6) and (5,6).
3. The sector of a circle of diameter π cm has an area of 2π cm. Determine the exact values of the sector angle and the arc length of the sector.
4. Determine the general solution to $\sin(2x)+\cos(x)=0$ both algebraically and graphically. Use exact values where possible; otherwise, give approximate measures to the nearest thousandth.
5. Prove the following trigonometric identities algebraically:

   a) $(\sin x+\cos x)^2-\sin 2x=1$ b) $\dfrac{\cos ø+\sin ø}{\cos ø-\sin ø}=\dfrac{1+\sin 2ø}{\cos 2ø}$

EXPONENTS AND LOGARITHMS

1. Express as a single logarithm: $3+\frac{1}{2}\log_2 a-3\log_2 b$
2. Determine an earthquake 800 times more intense than an earthquake of 6.3 on the Richter scale.
3. Describe the transformations that transform the first function to the second in each below.

   a) $y=4^x$, $y=-2^{(4-2x)}+3$ b) $y=\log_3 x$, $y=1-2\log_3\left(\frac{x}{5}\right)$

4. Solve algebraically over the set of real numbers: $\log(8x+4)=1+\log(x+1)$
5. A pump reduces the air pressure in a large tank by 50% every 12 seconds. Determine an exponential function to model this situation, where $P(t)$ is the pressure after t seconds and P_0 is the starting pressure. How long will it take for the air pressure in the tank to be reduced to 1% of its starting pressure?

COMBINATORICS

1. How many different ways can 5 people sit on a bench if one particular single person must sit at an end and one couple must sit together?
2. In a standard deck of 52 playing cards, there are 13 denominations of cards. Each of these denominations has four different suits (♠ ♣ ♥ ♦). How many different 5-card hands are possible that contain at most 1 heart (♥)?
3. From a group of 10 females and 12 males, a committee of 5 people is selected. How many different possible committees have at least 1 member of each sex represented?
4. In the expansion of $(2x^2 - 3)^8$, determine the middle term.
5. How many even integers exist between 111 and 7999? Compare using the fundamental counting principle to a more elementary counting method. Which do you prefer and why?