

Name: _____

Date: _____

Trigonometry Assignment

Choose any number of questions to complete so that you total 20 marks. Extra questions will not be marked; however, if you give an *honest attempt* at all questions, you will get a bonus mark toward your Trigonometry test. The rules: If you choose a question, you must complete every part of it. Clearly indicate question number and final answer (if appropriate) in your solutions. Please show all work for full marks. Use exact values whenever possible. Due the class before the test. (20 marks of a possible 35 marks to choose from)

1. A sector of a circle with radius 5 cm has an arc of length 15 cm. Determine the area of the sector. (1 mark)
2. Suppose P is the function that maps the angle θ in standard position, measured in radians, to the point (x, y) on the unit circle, i.e. $P(\theta) = (x, y)$ as discussed in class. Now suppose $P(\theta)$ lies at the intersection of the unit circle and the line joining the point $A(4, 3)$ to the origin. (3 marks)
 - a) Determine the exact coordinates of $P(\theta)$. [hint: see #18 on p. 189 in text]
 - b) Determine the exact coordinates of $P(\pi - \theta)$.
 - c) Determine the equation of the circle centred at the origin that passes through point A .
3. The point $(-5, 6)$ lies on the terminal arm of an angle θ in standard position and $\theta > 0$. Determine the exact values of the six trigonometric ratios for angle θ . (2 marks)
4. Find the exact value of $\csc \frac{-2\pi}{3}$. (1 mark)
5. Consider the function $y = \cos\left(\frac{\pi}{2} \sin x + \pi\right)$. (2 marks)
 - a) Determine the range of the function analytically. Then confirm using graphing technology.
 - b) Write an equation of the form $y = a \cos b(x - c) + d$ to represent the graph of this function.
 - c) Write the original function as the composition of two sinusoidal functions. [optional]
6. Consider the function $y = 3 \cos(2x - \pi) + 5$. (3 marks)
 - a) Find the domain, range, amplitude, phase shift, vertical displacement, and period.
 - b) Sketch this function on grid paper over one or more cycles.
7. Determine the period and all asymptotes for $y = \tan(4x + \pi)$. (2 marks)
8. The function $d(t) = 3 \sin 0.17\pi(t - 1.5) + 8.5$ gives the depth of water, d , metres, at any time, t hours, during a certain day. A commercial shipping boat needs at least 6 m of water to dock safely. For how many hours in the 24 hour interval starting at $t = 0$ can the cruise ship dock safely (to the nearest hundredth)? (1 mark)
9. A Ferris wheel of diameter 30 m has its centre 17 m above the ground. It rotates once every 2 minutes. The ride starts at its lowest point. (4 marks)
 - a) Find the average angular speed of the wheel in radians per second.
 - b) How far does a rider travel if the ride lasts 6 min?
 - c) Determine an equation for the height above the ground, $h(t)$, of a rider at time t seconds.
 - d) How high above the ground, to the nearest tenth of a metre, would a rider be when the wheel has been in motion for 3.3 min?

10. Solve: $2\sin(3x)+1=0$, where $0 \leq x \leq 2\pi$. (3 marks)

11. For the function $2(1-\sin^2 x)+3\cos x+1=0$, determine all roots using exact values over the set of real numbers. (3 marks)

12. Solve to 4 decimal places: $x^2-1=\cos^2 x \sin x$, where $-\pi \leq x \leq \pi$. (1 mark)

13. Determine a sine equation that has the following general solution, where k is any integer:

$$k\pi, \frac{\pi}{6}+2k\pi, \frac{5\pi}{6}+2k\pi \quad (2 \text{ marks})$$

14. Simplify: $\frac{\tan(2\theta)\cos(2\theta)}{2\cos\theta}$ (2 marks)

15. Simplify: $\cos 15^\circ \sin 45^\circ - \sin 15^\circ \cos 45^\circ$ (1 mark)

16. Determine the exact value of $\tan \frac{11\pi}{12}$. (1 mark)

17. If $\cos A = \frac{3}{4}$, where angle A is in the 4th quadrant, find the exact value of $\sin 2A$. (1 mark)

18. Prove any number of trigonometric identities below: (2 marks each)

a) $\frac{\sec^2 x - 1}{\tan x} = \frac{\sin 2x}{1 + \cos 2x}$ b) $(\csc \theta - \sin \theta) \tan \theta = \frac{\sin 2\theta}{2\sin \theta}$ c) $\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} = 2 \tan^2 x$