Probability Homework Booklet
SOLUTIONS

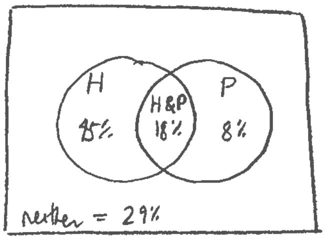
G ① $P(N) = \frac{1}{4}$ $P(\bar{N}) = 1 - P(N) = 1 - \frac{1}{4} = \frac{3}{4}$ ($\bar{N}$ is the complement of N)

C ② $\underset{1^{st}}{\frac{1}{6}} \cdot \underset{2^{nd}}{\frac{1}{6}} \cdot \underset{3^{rd}}{\frac{5}{6}}$

1st: 1 2 3 4 ⑤ 6 (1 of 6) 2nd: 1 2 3 4 5 ⑥ (1 of 6) 3rd: 1 ~~2~~ 3 4 5 6 (5 of 6)

We multiply to get prob of all 3 happening (recall we reserve addition for different cases)

B ③



$P(H \text{ or } P) = P(H) + P(P) - P(H \& P)$
$= 63\% + 26\% - 18\%$
$= 71\%$

$P(\text{neither}) = 1 - P(H \cup P)$ (complement ☺)
$= 100\% - 71\%$
$= 29\%$

(OR) Fill in Venn Diagram above: $P(H \text{ only}) = P(H) - P(H \& P)$
$= 63\% - 18\%$
$= 45\%$

$P(P \text{ only}) = P(P) - P(H \& P)$
$= 26\% - 18\%$
$= 8\%$

So $P(H \cup P) = P(H \text{ only}) + P(P \text{ only}) + P(H \& P)$
$= 45\% + 8\% + 18\%$
$= 71\%$

and $P(\text{neither}) = 1 - P(H \cup P)$
$= 100\% - 71\%$
$= 29\%$

The diagram with percentages in it are good enough for "showing work" i.e. you don't need to write all this out

D ④ $n = 9$
$r = 5$

Sample space (i.e. all possible outcomes) has ${}_9C_5 = 126$ outcomes

outcomes with exactly 3 girls is $\underbrace{{}_5C_3}_{\text{choose 3 of 5 girls}} \cdot \underbrace{{}_4C_2}_{\text{choose 2 of 4 boys to get the rest}} = 60$

So $P(\text{exactly 3 girls}) = \dfrac{\text{\# outcomes of exactly 3 girls}}{\text{\# outcomes in sample space}}$

$= \dfrac{60}{126}$

$\doteq 0.48$

C ⑤ A King is a face card and either can be a spade or heart. So only a spade and heart are mutually exclusive — a card can't be both a spade AND a heart! So S & H

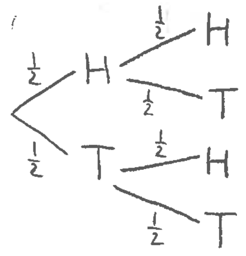
D ⑥ $n = 3+11+14$
$= 28$
$r = 2$
w/o replacement

Need to find prob of getting 2 black candies:

$$P(BB) = \underbrace{\left(\frac{14}{28}\right)}_{1^{st}} \underbrace{\left(\frac{13}{27}\right)}_{2^{nd}} = \underbrace{\frac{1}{2}}_{\text{reduced}} \cdot \frac{13}{27} = \frac{13}{54}$$

Thinking: on 1st pick we have 14 black candies available out of 28. Pick 1. Now we have 13 blacks left out of 27 candies left.

B ⑦



$P(HH) = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

$P(HT) = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

$P(TH) = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

$P(TT) = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

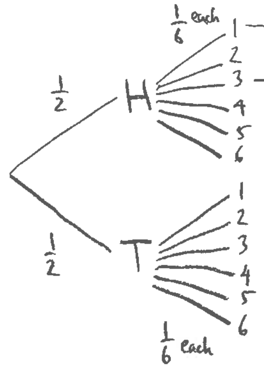
Let A be event of getting 2 tails
Let B be event of having at least 1 tail

Then $P(A) = P(TT) = \frac{1}{4}$
and $P(B) = P(HT) + P(TH) + P(TT) = \frac{3}{4}$

$$\text{So } P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

* P(A and B) = P(A) b/c for "getting 2 tails" AND "getting at least 1 tail" to occur, it means you're getting 2 tails! This is just P(A)

C ⑧



$\rightarrow P(H,1) = (\frac{1}{2})(\frac{1}{6}) = \frac{1}{12}$

$\rightarrow P(H,3) = (\frac{1}{2})(\frac{1}{6}) = \frac{1}{12}$

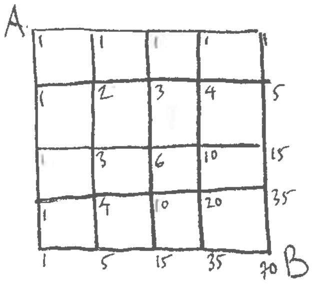
$P(H,1 \text{ or } H,3) = P(H,1) + P(H,3)$
$= \frac{1}{12} + \frac{1}{12}$
$= \frac{2}{12}$
$= \frac{1}{6}$

add different cases (we know they're diff cases b/c we can't get H,1 & H,3 at the same time)

Ⓞⓡ H,1 & H,2 are 2 outcomes from a sample space of 12

$\Longrightarrow P(H,1 \text{ or } H,3) = \frac{2}{12} = \frac{1}{6}$

D ⑨ Recall from last unit, there are ${}_8C_4 = 70$ ways to get from Bill's House (A) to the library (B):



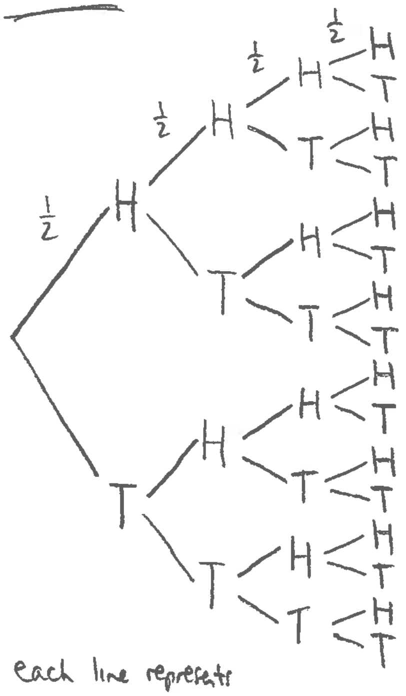
${}_{⑧}C_{④} = 70$

8 blocks from A to B, no matter which way you go

think 4 right or 4 down

The indicated route is only 1 of a total of 70 possible routes. So the probability that Bill chooses that one is $\frac{1}{70}$.

⑩ Method 1:



Sample space	Prob
HHHH	$(\frac{1}{2})^4 = \frac{1}{16}$
HHHT	$\frac{1}{16}$
HHTH	$\frac{1}{16}$
HHTT	$\frac{1}{16}$
HTHH	
HTHT	
HTTH	
HTTT	
THHH	
THHT	
THTH	
THTT	
TTHH	
TTHT	
TTTH	
TTTT	$\frac{1}{16} = P(\bar{N})$

✱ each line represents a probability of $\frac{1}{2}$
H = head T = tails

let N = Prob of getting at least one Head

Then $\bar{N}$ = Prob of getting No heads (or all tails)

So $P(N) = 1 - P(\bar{N})$

(we use the complement to find P(N))

$$\begin{aligned} P(N) &= 1 - P(\bar{N}) \\ &= 1 - P(TTTT) \\ &= 1 - \frac{1}{16} \\ &= \frac{15}{16} \end{aligned}$$

∴ prob of getting at least 1 Head is $\frac{15}{16}$

Note: this is why we don't need to draw the entire tree diagram, just the bottom branch containing all T's (compare to birthday problem in your notes)

Method 2:

Unlike the birthday problem, the probability for getting H or T is always the same each toss of the coin. This makes the prob and sample space very predictable. We can see that there are, for example, ${}_4C_3 = 4$ outcomes with exactly 3 H after 4 flips. And the probability would depend on the prob of getting a H vs a T (0.5 for each in this case ... see question 16). There is a formula we can use:

$$P(x) = {}_nC_x \, p^x q^{n-x} \quad , \text{ where } q = 1 - p$$

Prob of outcome x

# of trials in total

# of trials w/ outcome x

prob of not getting outcome x

prob of getting outcome x

In our case, we still use the complement (getting all tails ... see above) meaning $x = 4$ for 4 tails (could also use 0 for 0 heads) and $n = 4$ tosses of the coin. $p = \frac{1}{2}$ (prob of getting tails for each trial), so $q = 1 - \frac{1}{2} = \frac{1}{2}$ (prob of heads). So... $P(4) = {}_4C_4 (\frac{1}{2})^4 (\frac{1}{2})^0 = \frac{1}{16}$

$$\therefore P(\text{at least 1 head}) = 1 - \frac{1}{16} = \frac{15}{16}$$

(see #16 for another example)

D

⑪ Let A = event of student owning Playstation

B = ———— // ———— Nintendo Game Cube

Then $P(A) = 16\%$, $P(B) = 12\%$, $P(A \& B) = 5\%$.

$P(\text{neither } \underset{(\sim A)}{A} \text{ nor } \underset{(\sim B)}{B}) = 1 - P(A \text{ or } B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$

$= 16\% + 12\% - 5\%$

$= 23\%$

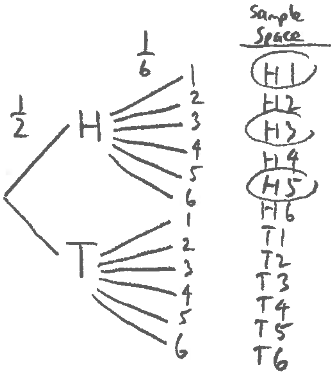
$\therefore P(\sim A, \sim B) = 100\% - 23\%$

$= 77\%$

Could draw Venn Diagram too

B

⑫



$P(H, \text{odd}\#) = \dfrac{\#\text{ outcomes of } H, \text{odd}\#}{\#\text{ outcomes in S.S.}}$

$= \dfrac{3}{12}$

$= \dfrac{1}{4}$

OR $P(H, \text{odd}\#) = P(H,1) + P(H,3) + P(H,5)$

$= \dfrac{1}{12} + \dfrac{1}{12} + \dfrac{1}{12}$

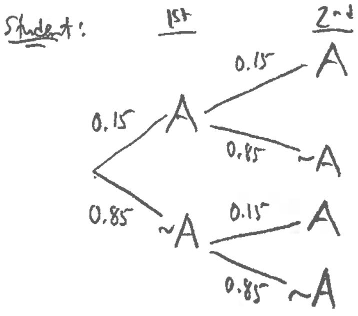
$= \dfrac{3}{12}$

$= \dfrac{1}{4}$

C

⑬ Let A = event that a gr 12 student plays basketball

~A = ———— // ———— doesn't play basketball



$P(AA) = (0.15)(0.15) = 0.0225$ — both play basketball

$P(A,\sim A) = (0.15)(0.85) = 0.1275$

$P(\sim A, A) = (0.85)(0.15) = 0.1275$

at least 1 plays basketball

$P(\sim A, \sim A) = (0.85)(0.85) = 0.7225$

can check ☺ → 1.0000

Given at least 1 plays bball, then in 1 out of 3 cases, both play b ball:

$$\frac{P(AA)}{P(AA) + P(A,\sim A) + P(\sim A, A)}$$

$$= \frac{0.0225}{0.0225 + 0.1275 + 0.1275}$$

$\doteq 0.081$ or 8.11%

OR Can use $P(A|B) = \dfrac{P(A \& B)}{P(B)}$, where A = event that both play b ball

B = event that at least 1 plays b ball

We can see that $P(A \underline{\underline{\&}} B)$ means $P(A)$ since A & B can only <u>both</u> occur when A & B = A

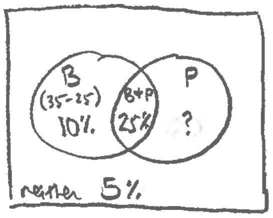
$\underset{\text{(both play)}}{P(A) = (0.15)^2 = 0.0225}$ $\quad P(B) = \underbrace{(0.15)(0.85)}_{\text{1st person plays}} + \underbrace{(0.85)(0.15)}_{\text{2nd person plays}} + \underbrace{(0.15)(0.15)}_{\text{both play}} = 0.2775$

$$\therefore P(A|B) = \frac{P(A \& B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{0.0225}{0.2775} \doteq 0.081 \text{ or } 8.11\%$$

Note: This "formula" generally works better as an idea than a plug in equation, as you can see above.

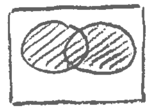
A ⑭ Section C appears to be $\frac{1}{8}$ of the circle, so that is the probability of landing on section C.

⑮



D

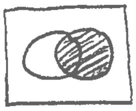
$P(B \text{ or } P) = 100 - 5 = 95$



$P(B \text{ or } P) = P(B) + P(P) - P(B+P)$

$\therefore\ 95 = 35 + P(P) - 25$

$\therefore\ P(P) = 85 \longrightarrow$



So Prob only liking peas is:

$P(P) - P(B+P)$

$= 85 - 25$

$= \underline{\underline{60\ \%}}$ or 0.6

Ⓞⓡ Fastest way: using Venn Diagram above: $P(\text{peas only}) = 100 - 10 - 25 - 5 = 60\%$ or 0.6

⑯ Use $P(x) = {}_nC_x\ p^x q^{n-x}$, where $q = 1 - p$

C

$n = 12$

$x = 8$

$p = 0.6$

$q = 1 - 0.6$

$= 0.4$

$$\underbrace{P(8)}_{\text{prob of getting exactly 8 successful putts (in hole)}} = {}_{\underbrace{12}_{\text{12 putts in all}}}C_{\underbrace{8}_{\text{get 8 in hole}}}\ \underbrace{(0.6)}_{\text{0.6 prob of getting putt in hole}}^{\overset{\text{\# in}}{8}}\ \underbrace{(0.4)}_{\text{0.4 prob of missing put}}^{12-8=4\ \text{\# misses}} \Longrightarrow P(8) = 0.212840939\underline{5}$$

$\doteq 0.21$

*note: you could use a tree diagram — but that's a lot of branches!

⑰ $n = 52$

$r = 3$

$P(\text{getting at least } 1 \diamond) = 1 - P(\text{getting No } \diamond\text{s})$ (using the complement)

C

$$\underbrace{{}_{13}C_0}_{\text{(choosing 0 of 13 }\diamond\text{s)}} \cdot \underbrace{{}_{39}C_3}_{\text{(choosing 3 cards from remaining)}} = 9\,139 \text{ 3-card combinations w/ No } \diamond\text{'s}$$

Total possible 3-card combinations is ${}_{52}C_3 = 22\,100$ (# outcomes in Sample Set)

$\therefore\ P(\text{getting No } \diamond\text{'s}) = \frac{9\,139}{22\,100} = 0.4135294118$ (this is $\frac{\text{\# outcomes w/ No }\diamond\text{s}}{\text{\# outcomes in S.S.}}$)

$\therefore\ P(\text{getting at least } 1 \diamond) = 1 - 0.4135294118 = 0.5864705882$ or about $\boxed{0.59}$

⑱ $P(\text{Head}) = \frac{1}{2}$

A $P(\text{rolling } 5) = \frac{1}{6}$

Since tossing a coin + rolling a die are independent events (ie one doesn't affect the other), we simply multiply:

$$P(H+5) = P(H)\,P(5) = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{6}\right) = \frac{1}{12}$$

⑲ $n = 10$ questions

$x = 7$ correct

B

$p = \frac{1}{4}$ (1 in 4 choices correct)

$q = 1 - \frac{1}{4} = \frac{3}{4}$ (prob guessing is incorrect)

use $P(x) = {}_nC_x \, p^x q^{n-x}$

$$\therefore P(7) = {}_{10}C_7 \left(\tfrac{1}{4}\right)^7 \left(\tfrac{3}{4}\right)^{10-7}$$

$$= {}_{10}C_7 \left(\tfrac{1}{4}\right)^7 \left(\tfrac{3}{4}\right)^3$$

⑳ $n = 8 + 10 = 18$

$r = 6$

C

outcomes w/ exactly 4 ♂ chosen: ${}_8C_4 \, {}_{10}C_2 = 3150$

(choose 4 ♂ of 8) (choose 2 ♀ of 10)

total outcomes possible (in Sample Space): ${}_{18}C_6 = 18\,564$

$\therefore$ prob of exactly 4 males chosen is $\dfrac{({}_8C_4)({}_{10}C_2)}{{}_{18}C_6} = \dfrac{3150}{18\,564} = 0.169683259 \cong 0.17$

㉑ Since this is w/o replacement and the events "1st card is a face card" & "2nd card is a queen" are NOT mutually exclusive, we have 2 cases to consider (so we add these cases)

A

Case 1: 1st card is not a queen:

$\left(\frac{8}{52}\right) \times \left(\frac{4}{51}\right)$

1st card: (J or K so $2 \times 4 = 8$ possibilities)

2nd card: (4 queens left in deck only 51 remaining)

Case 2: 1st card IS a queen:

$\left(\frac{4}{52}\right) \times \left(\frac{3}{51}\right)$

1st card: (4 queens to choose from in deck of 52)

2nd card: (3 queens left to choose from w/ only 51 cards left)

$\therefore$ prob 1st card is a face card & 2nd is a queen is $\left(\frac{8}{52}\right)\left(\frac{4}{51}\right) + \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = \dfrac{44}{2652} = \dfrac{11}{663}$

㉒ From tree diagram in question #10, we see there are 6 outcomes w/ exactly 2 heads (note ${}_4C_2 = 6$ ☺) out of a sample space of $2^4 = 16$ outcomes.

C

(long way)

So $P(\text{exactly } 2 H) = \frac{6}{16} = \frac{3}{8}$

(OR) use $P(x) = {}_nC_x \, p^x q^{n-x}$, where $q = 1 - p$

(fast way)

$n = 4$ flips

$x = 2$ heads

$p = \frac{1}{2}$ prob for H

$q = \frac{1}{2}$ prob for T

$$\therefore P(2) = {}_4C_2 \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2$$

$$= 6\left(\tfrac{1}{4}\right)\left(\tfrac{1}{4}\right)$$

$$= \tfrac{3}{8}$$

C ㉓ $\left(\frac{13}{52}\right)\left(\frac{12}{51}\right) = \frac{156}{2652} = \frac{1}{17}$ (13 ♡s of 52 cards for 1st card. Then 12 ♡s left of 51 cards left for 2nd card)

㉔ (choose Kim) (choose 2 of 4 left)

D $$\frac{{}_1C_1 \cdot {}_4C_2}{{}_5C_3} = \frac{6}{10} = \frac{3}{5}$$ $\left(\frac{\text{recall \# outcomes w/ Kim}}{\text{\# outcomes in Sample Space}}\right)$

(3 of 5 for S.S.)

Written Questions

① a) There are 3 outcomes w/ sum of 6: (4,2), (3,3), (2,4) & 16 outcomes in the Samplespace (see chart)

∴ prob is $\frac{3}{16}$

b) multiple of 3 for product in 7 outcomes: (1,3), (3,1), (2,3), (3,2), (3,3), (3,4), (4,3) & 16 outcomes in S.S.

∴ prob is $\frac{7}{16}$

c) (2,1), (3,1), (3,2), (4,1), (4,2), (4,3) $\Longrightarrow \frac{6}{16}$

d) $\frac{3}{16} + \frac{7}{16} - \frac{1}{16} = \frac{9}{16}$ (can use P(A or B) = P(A) + P(B) − P(A & B))

(1 outcome in common)

e) 1 in common

\# outcomes w/ 1st die 4: (4,1), (4,2), (4,3), (4,4) (event A)

\# outcomes w/ sum dice of 6: (4,2), (3,3), (2,4) (event B)

$$P(A|B) = \frac{P(A \& B)}{P(B)} = \frac{\text{\# outcome w/ 1st die 4 \& sum of dice of 6}}{\text{\# outcomes w/ sum dice of 6}} = \frac{1}{3}$$

② a) $$\frac{{}_{13}C_5 \cdot {}_{39}C_0}{{}_{52}C_5} = \frac{1287}{2598960} \cong 0.0004952$$

(13 ♤ in deck, choose 5; choose 0 of 39 other kinds
w/ ${}_{52}C_5$ cards in SS

b) $$\frac{{}_{13}C_2 \cdot {}_{13}C_2 \cdot {}_{26}C_1}{{}_{52}C_5} = \frac{158184}{2598960} \cong 0.06086$$

(choose 2 of 13 ♡; choose 2 of 13 ♤; choose 1 of remaining 26 cards. SS has ${}_{52}C_5$ card combinations)

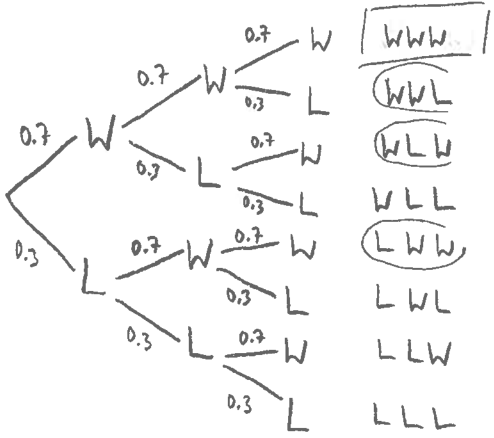
③ use $P(x) = {}_nC_x\, p^x q^{n-x}$, $q = 1-p$ $n = 3$ games; $p = 0.7$ (prob of winning) so $q = 0.3$ (prob of losing)

a) $x=3$ $P(3) = {}_3C_3\,(0.7)^3(0.3)^0 = 0.343$

b) $x=2$ or $x=3$ $P(2 \text{ or } 3) = P(2) + P(3) = {}_3C_2\,(0.7)^2(0.3)^1 + 0.343 = 0.441 + 0.343 = 0.784$

c) $P(3) \div P(2 \text{ or } 3) = 0.343 \div 0.784 = 0.4375$

(OR) use Tree diagram



a) $P(WWW) = (0.7)^3 = 0.343$

b) $$\begin{aligned} P(\text{at least } 2W) &= P(WWL) + P(WLW) + P(LWW) + P(WWW) \\ &= (0.7)^2(0.3) + (0.3)(0.7)(0.3) + (0.3)(0.7)^2 + (0.7)^3 \\ &= 3(0.7)^2(0.3) + (0.7)^3 \\ &\left(= {}_3C_2\,(0.7)^2(0.3)^1 + {}_3C_3\,(0.7)^3(0.3)^0\right) \\ &= 0.784 \end{aligned}$$

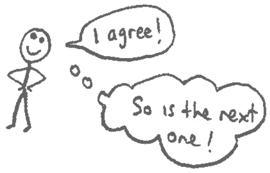
c) If you win at least twice means given that you win twice

So use $P(A|B) = \dfrac{P(A + B)}{P(B)}$ where A = 3 wins
B = at least 2 wins

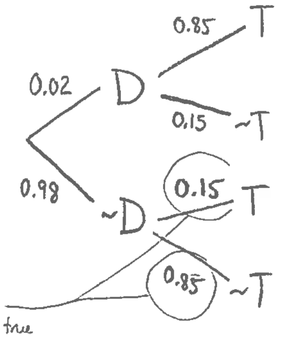
We can see that $P(A+B) = P(A)$ since having 3 wins & at least 2 wins means you won 3 times

$$\therefore\ P(A|B) = \frac{P(A)}{P(B)} = \frac{0.343}{0.784} = 0.4375$$

✳ This is a perfect test question!



④ Let D represent population w/ diabetes. Then ~D are those w/o diabetes.
Let T represent a positive test result. Then ~T is not positive (considered negative in question).



$P(D+T) = (0.02)(0.85) = 0.017$

$P(D+\sim T) = (0.02)(0.15) = 0.003$

$P(\sim D+T) = (0.98)(0.15) = 0.147$

$P(\sim D+\sim T) = (0.98)(0.85) = 0.833$

Careful!
0.85 that test is true
0.15 —"— false

Need to find prob of person who tests negative (given they test ~T) does not have diabetes (is ~T & ~D)

$$P(\sim T) = \overset{0.003}{P(D+\sim T)} + \overset{0.833}{P(\sim D+\sim T)} = 0.836$$

$$P(\sim D+\sim T) = 0.833$$

$$\text{Then } P(\sim D+\sim T \,|\, \sim T) = \frac{P(\sim D+\sim T)}{P(\sim T)} = \frac{0.833}{0.836} \cong 0.99641 \text{ or about } 99.64\%$$

⑤ $$\begin{aligned} P(\text{@ least 1 makes shot}) &= 1 - P(\text{neither make the shot}) \\ &= 1 - \left(\tfrac{1}{3}\right)\left(\tfrac{2}{5}\right) \\ &= \frac{13}{15} \end{aligned}$$

$P(\text{Cole makes shot}) = \frac{2}{3} \Rightarrow P(\text{Cole doesn't}) = \frac{1}{3}$

$P(\text{Amanda makes shot}) = \frac{3}{5} \Rightarrow P(\text{Amanda doesn't}) = \frac{2}{5}$

They are independent of one another (so we can multiply)

(OR) Cases: $$P(\text{@ least 1 makes shot}) = \underbrace{\left(\tfrac{2}{3}\right)\left(\tfrac{3}{5}\right)}_{\text{(both make it)}} + \underbrace{\left(\tfrac{1}{3}\right)\left(\tfrac{3}{5}\right)}_{\text{(just Am)}} + \underbrace{\left(\tfrac{2}{3}\right)\left(\tfrac{2}{5}\right)}_{\text{(just Cole)}} = \frac{13}{15}$$

⑥ This question screams "CASES!"

a) <u>Case 1</u>: a 1 or 2 is rolled (Jar A): $\left(\frac{2}{6}\right)\left(\frac{7}{12}\right) = \frac{14}{72}$

<u>Case 2</u>: a 3, 4, 5, or 6 is rolled (Jar B): $\left(\frac{4}{6}\right)\left(\frac{4}{12}\right) = \frac{16}{72}$

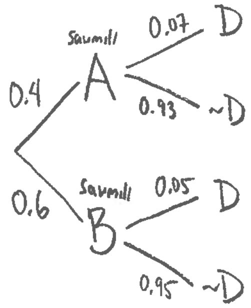
add: $\frac{30}{72}$ or $\frac{5}{12}$

$\frac{\text{\# fitting criterion}}{\text{6 possible \#'s on die}}$ $\frac{\text{\# white}}{\text{\# balls}}$

b) $$P(\text{white from A} \mid \text{white selected}) = \frac{P(\text{white from A \& white})}{P(\text{white})} = \frac{P(\text{white from A})}{P(\text{white})} = \frac{\left(\frac{14}{72}\right)}{\left(\frac{30}{72}\right)} = \frac{14}{30} = \frac{7}{15}$$

∵ white from A is a subsumed in white selected

⑦



$P(A \& D) = (0.4)(0.07) = 0.028$

$P(A \& \sim D) = (0.4)(0.93) = 0.372$

$P(B \& D) = (0.6)(0.05) = 0.03$

$P(B \& \sim D) = (0.6)(0.95) = 0.57$

If a randomly picked board is discoloured, (ie, <u>given</u> it is discoloured: D)
what is the prob it came from Sawmill A
(i.e. it is D & A)

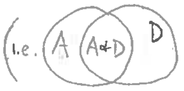
$P(D) = P(A \& D) + P(B \& D) = 0.058$

& we know $P(A \& D) = 0.028$

D = discoloured
~D = not discoloured

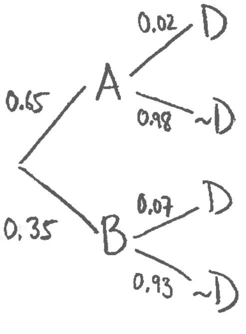
$$\therefore P(A \& D \mid D) = \frac{P(A \& D)}{P(D)} = \frac{0.028}{0.058} \cong 0.4823$$ or about 48%

* again (A & D) & D is redundant ⟶ so we use A & D (i.e.



A & D is a subset of D)

⑧ D = defective pin
~D = not defective



By now, you may be used to these types of questions & simply do:

$$\frac{P(A \& D)}{P(A \& D) + P(B \& D)} = \frac{(0.65)(0.02)}{(0.65)(0.02) + (0.35)(0.07)} = \frac{0.013}{0.0375} = 0.34\bar{6}$$ or about 35%

If not, analyze: $\underbrace{\text{"What is the probability that a defective pin}}_{\text{given that a pin is defective, (2 cases: A \& D or B \& D)}}$ $\underbrace{\text{comes from factory A?"}}_{\text{what is the prob it comes from A (1 case: A \& D b/c we're told it's defective \& from A)}}$

* take another look at #4 & #7 now to see if you can do them more efficiently