# Probability Review Assignment Full Solutions

(1) Two cases: 4 white + 6 black

C

i) draw 2 whites $\longrightarrow$ $P(2 \text{ whites}) = P(1^{st} \text{ is white}) \cdot P(2^{nd} \text{ is white})$

$$= \left(\frac{\#\text{ white balls}}{\text{total }\#\text{ balls}}\right)\left(\frac{\#\text{ white balls left}}{\text{total }\#\text{ balls left}}\right)$$

$$= \left(\frac{4}{10}\right)\left(\frac{3}{9}\right)$$

$$= \frac{12}{90}$$

ii) draw 2 blacks $\longrightarrow$ $P(2 \text{ blacks}) = P(1^{st} \text{ is black}) \cdot P(2^{nd} \text{ is black})$

$$= \left(\frac{\#\text{ black}}{\text{total}}\right)\left(\frac{\#\text{ black left}}{\text{total left}}\right)$$

$$= \left(\frac{6}{10}\right)\left(\frac{5}{9}\right)$$

$$= \frac{30}{90}$$

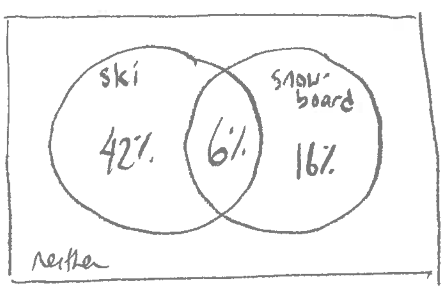
$\therefore$ $P(2 \text{ balls drawn are same colour}) = \frac{12}{90} + \frac{30}{90}$

$$= \frac{42}{90}$$

$$= \frac{7}{15}$$

(2)

C




Venn Diagram

$P(\text{Only snowboard}) = P(\text{snowboard}) - P(\text{both})$

$$= 22\% - 6\%$$

$$= 16\%$$

$P(\text{Only ski}) = P(\text{ski}) - P(\text{both})$

$$= 48\% - 6\%$$

$$= 42\%$$

$P(\text{neither ski nor snowboard}) = 100\% - (42\% + 6\% + 16\%)$

$$= 36\%$$

OR

let event A = ski

event B = snowboard

Then $P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$

$$= 48\% + 22\% - 6\%$$

$$= 64\%$$

and $P(\overline{A \text{ or } B}) = 100\% - 64\% = 36\%$

③ B

| 1st Coin | 2nd Coin | Prob |
|---|---|---|
| H (0.5) | H (0.5) | $P(HH) = (0.5)(0.5) = 0.25$ |
| | T (0.5) | $P(HT) = (0.5)(0.5) = 0.25$ |
| T (0.5) | H (0.5) | $P(TH) = (0.5)(0.5) = 0.25$ |
| | T (0.5) | $P(TT) = (0.5)(0.5) = 0.25$ |

Check: 1.00 ✓

OR

4 possible outcomes:

HH → $P(HH) = \frac{1}{4}$

HT → $P(HT) = \frac{1}{4}$

TH → $P(TH) = \frac{1}{4}$

TT → $P(TT) = \frac{1}{4}$

check: $\frac{4}{4} = 1$ ✓

Prob both heads given at least 1 is head:

outcomes w both heads: HH

outcomes w at least 1 head: HH, HT, TH

$$\therefore P(HH \mid \text{at least } 1H) = \frac{P(HH)}{P(\text{at least } 1H)} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}} = \frac{1}{3}$$

note $P(HH \text{ \& at least } 1H)$ is just $P(HH)$... why

OR $= \frac{\#\text{ cases } HH}{\#\text{ cases at least } 1H} = \frac{1}{3}$

quick & easy way

④ C

Sum is odd if one is even & other is odd. That is, when 1st spinner is even, which is $\frac{2}{3}$ of the time since there are 2 evens & 1 odd.

$\therefore P(\text{sum is odd}) = \frac{2}{3}$

⑤ In this case, order matters in that the # possible outcomes is dependent on when you obtain the 2 fives.

long, but intuitive, way

So... $P(\text{exactly } 2\ 5\text{'s}) = \frac{\#\text{ outcomes w exactly } 2\ 5\text{'s}}{\#\text{ total outcomes}}$

$\#\text{ total outcomes} = \underset{1st}{6} \times \underset{2nd}{6} \times \underset{3rd}{6} \times \cdots \times \underset{8th}{6} = 6^8$ ∵ 6 possibilities for each roll

$\#\text{ outcomes w exactly } 2\ 5\text{'s} = \underbrace{\underset{\substack{1st\\(5)}}{1} \times \underset{\substack{2nd\\(5)}}{1} \times \underset{3rd}{5} \times \underset{4th}{5} \times 5 \times 5 \times 5 \times \underset{8th}{5}}_{\text{but there are } {}_8C_2 \text{ ways to place the 5's}} = 5^6$ ∵ 5 options left for other dices (can't repeat #5)

this is why: $\underset{1st\ 5}{\frac{1}{\downarrow}}$ $\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{7 \text{ options for 2nd } 5}$

1st 5 — 6 options for 2nd 5 … 5 options … 1 option

C

So we get $7+6+5+4+3+2+1 = 28$ options or ${}_8C_2$ options

$\Longrightarrow 28 \times 5^6$ outcomes

$\therefore P(\text{exactly } 2\ 5\text{'s}) = \dfrac{28 \times 5^6}{6^8} \doteq 0.26$

OR Simply $({}_8C_2)\left(\frac{1}{8}\right)^2\left(\frac{5}{6}\right)^6 \doteq 0.26$

2 5's — 5 of 6 #'s for 6 others

Picking 2 spots of 8 but we divide by 2! b/c 2 repeats (both 5 — repetition!)

So $\dfrac{{}_8P_2}{2!} = {}_8C_2$

2 5's

FAST WAY!

✱ see formula sheet: $P(x) = {}_nC_x\, p^x q^{n-x}$, $q = 1-p$

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⑥ a) $P(1 \text{ club}) = \dfrac{\#\text{ outcomes w/ 1 club}}{\#\text{ total outcomes}}$

$= \dfrac{{}_{13}C_1 \cdot {}_{39}C_4}{{}_{52}C_5}$

13 clubs in deck; choose 1 — 39 non clubs; choose 4

$= \dfrac{1\,069\,263}{2\,598\,960}$

$\doteq 0.4114$ or about 41 %

b) $P(\text{at most 1 club}) = P(\text{no clubs}) + P(1 \text{ club})$

we know $P(1 \text{ club}) \doteq 0.4114$

find $P(\text{no clubs})$ … $P(\text{no clubs}) = \dfrac{{}_{13}C_0 \cdot {}_{39}C_5}{{}_{52}C_5} \doteq 0.2215$

$\therefore P(\text{at most 1 club}) \doteq 0.4114 + 0.2215$

$\doteq 0.63$ or about 63 %

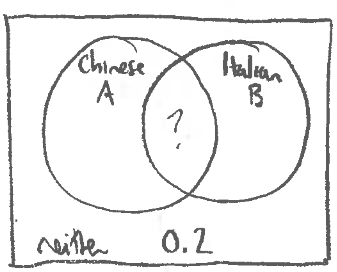
⑦ $P(\text{red ace}) = \dfrac{\#\text{ red aces}}{\text{total \# cards}}$

A

$= \dfrac{2}{52}$ ← ace of ♡ + ace of ◇

$= \dfrac{1}{26}$

⑧




C

use $P(A \text{ or } B) = 1 - P(\overline{A \text{ or } B}) = 1 - 0.2 = 0.8$

& $P(A \text{ or } B) = P(A) + P(B) - P(A \,\&\, B)$

$\therefore P(A \,\&\, B) = P(A) + P(B) - P(A \text{ or } B)$

$= 0.7 + 0.22 - 0.8$

$= 0.12$ or 12 (out of 100 people)

(9) $P(\text{Flyers win}) = \frac{5}{9} \Rightarrow P(\text{Flyers lose}) = \frac{4}{9}$

$P(\text{Bears win}) = \frac{12}{17} \Rightarrow P(\text{Bears lose}) = \frac{5}{17}$

B

So $P(\text{Flyers win} \;\&\; \text{Bears lose}) = P(\text{Flyers win}) \cdot P(\text{Bears lose})$ (can multiply ∵ independent)

$$= \frac{5}{9} \cdot \frac{5}{17}$$

$$= \frac{25}{153}$$

(10) $n = 10$, $r = 5$ ; 4 red, 6 black

$P(\text{at least 3 red balls}) = P(\text{3 red balls}) + P(\text{4 red balls})$

C

$$= \frac{{}_4C_3 \cdot {}_6C_2}{{}_{10}C_5} + \frac{{}_4C_4 \cdot {}_6C_1}{{}_{10}C_5}$$

$$= \frac{60}{252} + \frac{6}{252}$$

$$= \frac{66}{252}$$

$$\doteq 0.2619$$

Compare with #1

(11)

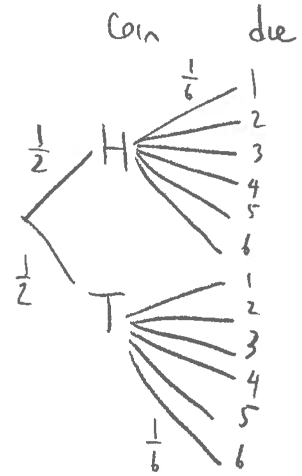
| Die (or bag) | Ball | prob |
|---|---|---|
| $\frac{2}{6} = \frac{1}{3}$ 1 or 2 (or bag A) | $\frac{2}{7}$ G | $P(AG) = \left(\frac{1}{3}\right)\left(\frac{2}{7}\right) = \frac{2}{21}$ |
| | $\frac{5}{7}$ W | $P(AW) = \left(\frac{1}{3}\right)\left(\frac{5}{7}\right) = \frac{5}{21}$ |
| $\frac{4}{6} = \frac{2}{3}$ 3,4,5, or 6 (or bag B) | $\frac{4}{7}$ G | $P(BG) = \left(\frac{2}{3}\right)\left(\frac{4}{7}\right) = \frac{8}{21}$ |
| | $\frac{3}{7}$ W | $P(BW) = \left(\frac{2}{3}\right)\left(\frac{3}{7}\right) = \frac{6}{21}$ |

a) $P(W) = P(AW) + P(BW) = \frac{5}{21} + \frac{6}{21} = \frac{11}{21}$ or about 52%

b) $P(A|W) = \frac{P(AW)}{P(W)} = \frac{\frac{5}{12}}{\frac{11}{12}} = \frac{5}{11}$ or about 45%

(12) Sample Space:

$P(H5) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) = \frac{1}{12}$ $\because$ flipping coin + rolling die are independent




(A)

(13) $_{10}C_7 \left(\frac{1}{4}\right)^{7}\left(\frac{3}{4}\right)^{3}$

7 correct

3 incorrect

$\frac{1}{4}$ prob of guessing correctly

$\frac{3}{4}$ prob of guessing wrongly

of the 10 questions, we're choosing 7 to be correct

(formula $P(x) = {}_nC_x\, p^x q^{n-x}$, $q = 1-p$)

(D)

(14) $n = 18$   $8 \male$

$r = 6$   $10 \female$

$$P(4\male) = \frac{\#\text{ outcomes}}{\text{total \# outcomes}}$$

$$= \frac{{}_8C_4 \cdot {}_{10}C_2}{{}_{18}C_6}$$

$$= \frac{3150}{18564}$$

$$\doteq 0.17$$

(C)

(15) $P(\text{face card}\,;\,\text{queen}) = P(\text{queen}\,;\,\text{queen}) + P(\underset{(J,\,K)}{\text{face card except queen}}\,;\,\text{queen})$

$$= \left(\frac{4}{52}\right)\left(\frac{3}{51}\right) + \left(\frac{8}{52}\right)\left(\frac{4}{51}\right)$$

$$= \frac{44}{2652}$$

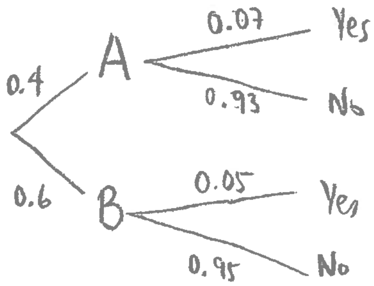
$$= \frac{11}{663}$$

not mutually exclusive!

(so do these 2 cases)

(A)

(16) Sawmill    discolored    Probability




$P(A; Yes) = (0.4)(0.07) = 0.028$

$P(A; No) = (0.4)(0.93) = 0.372$

$P(B; Yes) = (0.6)(0.05) = 0.03$

$P(B; No) = (0.6)(0.95) = 0.57$

D

$$P(A \mid Yes) = \frac{P(A; Yes)}{P(Yes)} = \frac{0.028}{0.058} \doteq 0.4828$$ or about 48%

$$P(Yes) = P(A; Yes) + P(B; Yes) = 0.028 + 0.03 = 0.058$$

(17) $(0.6)^{8}(0.4)^{4}\left({}_{12}C_{8}\right) \doteq 0.21$

C

0.6 prob of being successful 8 times

0.4 prob of missing 4 times

Of the 12 holes, we choose 8 to be successful in

Compare w/ #5 & #13

(18) $P(\text{@least 1 Diamond}) = 1 - P(\text{no diamonds})$

$$P(\text{no diamonds}) = \frac{{}_{13}C_{0} \cdot {}_{39}C_{3}}{{}_{52}C_{3}}$$

C

13 diamonds; choose none

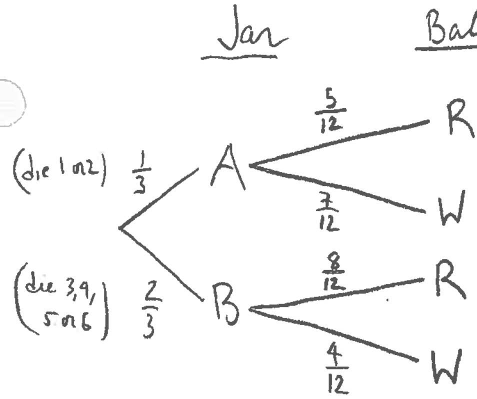
39 nondiamonds (other cards); choose 3

52 cards in total; choose 3 (total # outcomes)

$$= \frac{9139}{22100} \doteq 0.4135$$

$\therefore P(\text{@least 1 diamond}) \doteq 1 - 0.4135 \doteq 0.59$ or about 59%

(19) (like #11 ... so look at that sol^n 1st to try #19 ... if that fails, look below ☺)




Prob

$P(AR) = \left(\frac{1}{3}\right)\left(\frac{5}{12}\right) = \frac{5}{36}$

$P(AW) = \left(\frac{1}{3}\right)\left(\frac{7}{12}\right) = \frac{7}{36}$

$P(BR) = \left(\frac{2}{3}\right)\left(\frac{8}{12}\right) = \frac{16}{36}$

$P(BW) = \left(\frac{2}{3}\right)\left(\frac{4}{12}\right) = \frac{8}{36}$

a) 
$$\begin{aligned} P(W) &= P(AW) + P(BW) \\ &= \frac{7}{36} + \frac{8}{36} \\ &= \frac{15}{36} \\ &= \frac{5}{12} \text{ or } 0.41\bar{6} \text{ or } 41.\bar{6}\% \end{aligned}$$

b) 
$$\begin{aligned} P(A|W) &= \frac{P(AW)}{P(W)} \\ &= \frac{\frac{7}{36}}{\frac{15}{36}} \\ &= \frac{7}{15} \text{ or } 0.4\bar{6} \text{ or } 46.\bar{6}\% \end{aligned}$$

(20) a) Experimental probability $= \dfrac{\text{\# outcomes we're look for in experiment}}{\text{total \# trials done}}$ 〉 over many trials, we expect the experimental prob to be close to the theoretical prob (we've been doing theor. prob. in class)

$$\begin{aligned} P(HHH) &= \frac{\text{\# trials w/ HHH}}{\text{total \# trials}} \\ &= \frac{2}{10} \\ &= \frac{1}{5} \text{ or } 0.2 \text{ or } 20\% \end{aligned}$$

b) 
$$\begin{aligned} P(HHH) &= \left(\frac{1}{2}\right)^3 \\ &= \frac{1}{8} \text{ or } 0.125 \text{ or } 12.5\% \end{aligned}$$

c) use $P(x) = {}_nC_x\, p^x q^{n-x}$ ... so $P(2 \text{ HHH in 10 trials}) = {}_{10}C_2 \left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^8$

where $q = 1 - p$   $n = 10$, $x = 2$   $\doteq 0.242$