Transformations Homework Booklet — Full solutions for Multiple Choice

B ① $y=\sqrt{x}$ moved 4 right $\Longrightarrow$ $y=\underbrace{\sqrt{x-4}}$
(x-values)
x-values — opposite of what you think: 4 right $\Rightarrow$ -4

C ② $f(x)=5x-1 \Longrightarrow x\in\mathbb{R}$ + $f(x)\in\mathbb{R}$ (b/c this is a line)
let $y=f(x)=5x-1$ then switch $y\leftrightarrow x$: $x=5y-1$

$x+1=5y$ (add 1 to both sides)

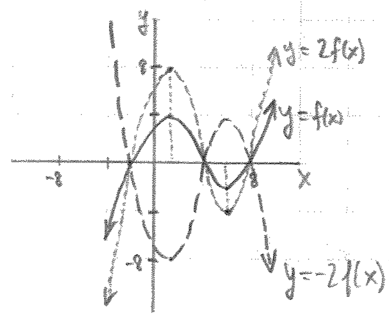
$\frac{x+1}{5}=y$ (divide both sides by 5)

$\therefore f^{-1}(x)=\frac{x+1}{5}$ (no restrictions)

B ③ $y=f(x) \longrightarrow y=\overset{\text{affects y-values}}{-2}f(x)$

reflection in x-axis (y-values change sign)

y-values double (on graph)



with table of values:

$y=f(x)$

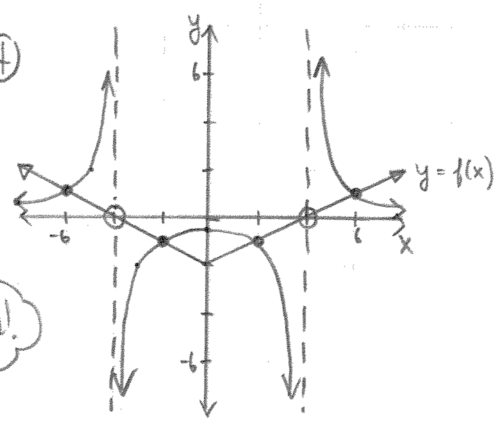
x	y
-2	0
1	4
4	0
5	-2
8	0

$y=-2f(x)$

x	y
-2	$-2(0)=-2$
1	$-2(4)=-8$
4	$-2(0)=0$
5	$-2(-2)=4$
8	$-2(0)=0$

So plot $(-2,-2)$; $(1,-8)$; $(4,0)$; $(5,4)$; $(8,0)$ above and connect with curve

D ④



Symmetrical!

• mark values of $y=f(x)=\pm1$ (invariant points)
o mark values of $y=f(x)=0$ (vertical asymptotes)

why? $f(x)=\pm1 \Rightarrow \frac{1}{f(x)}=\frac{1}{\pm1}=\pm1$

$f(x)=0 \Rightarrow \frac{1}{f(x)}=\frac{1}{0}$ which is undefined

if all else fails:

$y=f(x)$

x	y
-8	2
-5	$\frac{1}{2}$
-3	$-\frac{1}{2}$
0	-2

$y=\frac{1}{f(x)}$

x	y
-8	$\frac{1}{(2)}=\frac{1}{2}$
-5	$\frac{1}{(\frac{1}{2})}=2$
-3	$\frac{1}{(-\frac{1}{2})}=-2$
0	$\frac{1}{(-2)}=-\frac{1}{2}$

use symmetry for positive x-values

A ⑤ $y = x^3 + x^2 \longrightarrow$ reflected in y-axis

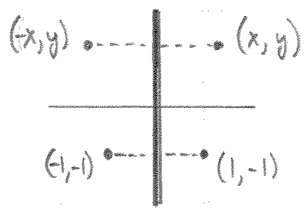
let $f(x) = x^3 + x^2$ then $\underbrace{y = f(-x)}_{\text{formula for reflection in y-axis}} = (-x)^3 + (-x)^2$

$= (-x)(-x)(-x) + (-x)(-x)$

$= -x^3 + x^2$

$\therefore\ y = -x^3 + x^2$

A ⑥ $y = f(-x)$ represents reflection of $y = f(x)$ in y-axis. Notice negative <u>in brackets</u>, so it negates all x-values, leaving y-values fixed as in diagram.



A ⑦ $y = \frac{1}{7} f(x)$ has been <u>vertically</u> compressed by factor $\frac{1}{7}$

outside of "f(x)" so affects y-values (i.e. vertical). Every y-value is multiplied by $\frac{1}{7}$ (to compress graph vertically).

A ⑧ let $y = f(x) = x^3 - 27$ then switch $x \leftrightarrow y$ to find inverse $y = f^{-1}(x)$:

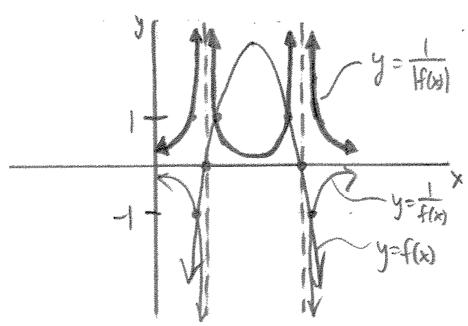
$x = y^3 - 27$

$x + 27 = y^3$ (add 27 to LS & RS)

$\sqrt[3]{x+27} = y$ (take 3rd root of LS & RS)

$\therefore\ f^{-1}(x) = \sqrt[3]{x+27}$ (no restrictions — go + test $x = -28$ ☺)

A ⑨



• mark V. asymptote @ $f(x) = 0$ and fixed points @ $f(x) = \pm 1$ (aka invariant points)

Don't forget LAST step:

<u>notice</u>: $y = f(x)$ + $y = \frac{1}{f(x)}$ always on same side of the x-axis in the given interval } then flip for $y = \frac{1}{f(x)} < 0$ in x-axis!! ☺

B ⑩ $y = f(2x + 10)$

$= f[2(x + 5)]$

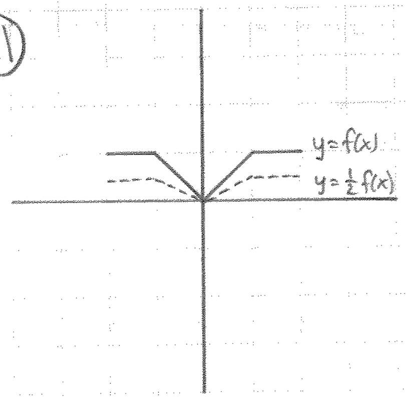
multiplicative then additive

$y = f(x) \longrightarrow y = f(2x) \longrightarrow y = f[2(x+5)]$

point on graph: opposite! deals with x so divide by 2; opposite to x value so subtract by 5

$(4, -3) \longrightarrow (\frac{4}{2}, -3) = (2, -3) \longrightarrow (2-5, -3) = \underline{(-3, -3)}$

D (11)



$y = \frac{1}{2} f(x)$

- outside of "f(x)" $\Rightarrow$ affects y values only
- $\frac{1}{2} \times f(x)$ suggests multiplication of y values by $\frac{1}{2}$

* recall "f(x)" represents original y-values since $y = f(x)$

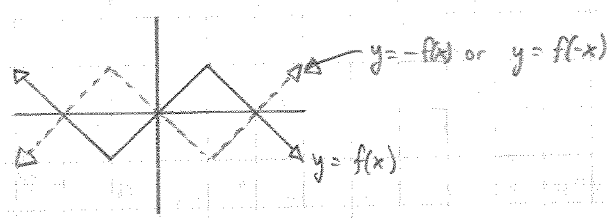
C (12) $y = f(x) \longrightarrow y = f(\underbrace{x-2}_{\text{x-values}}) \underbrace{+3}_{\text{y values}}$

opposite! +2   +3

$(a,b) \longrightarrow (a+2, b+3)$

C (13) The vertical asymptotes where $y = f(x) = 0$ are a clear sign of $y = \frac{1}{f(x)}$. So are the invariant points at (0, 1); (2, -1); (4, 1) and reciprocal y-values of the fn on the right compared to $y = f(x)$.

B (14) $f(-x) = -f(x)$ means $y = f(-x)$ and $y = -f(x)$; that is, we can reflect $y = f(x)$ into the y-axis or x-axis and we'd get the same graph (not necessarily same as $y = f(x)$ though). Note this is true for B — reflect it in the x-axis & y-axis to see ☺



A (15) Notice we have a reflection in the y-axis ie $y = f(-x)$ followed by a translation 2 units left to get $y = f[-(x+2)]$

So $y = f(\underbrace{-1}_{a}(x - \underbrace{(-2)}_{b})) \Rightarrow a = -1, b = -2$

A (16) $y = \sqrt{\underbrace{x-3}_{\text{right 3}}} \underbrace{+1}_{\text{up 1}}$

B (17) See #2

$y = 3x + 2$
$x = 3y + 2$
$x - 2 = 3y$
$y = \frac{x-2}{3}$

$\therefore f^{-1}(x) = \frac{x-2}{3}$

Lots of "repeats" coming... see similar questions for complete explanations.

A (18) reflect in y-axis: $y = f(-x) \implies$ x values negated:

$$5-x = 2y^2 + y \longrightarrow 5-(-x) = 2y^2+y$$
$$5 + x = 2y^2 + y$$

D (19) $y = \underbrace{3}_{2^{nd}}\underbrace{|f(x)|}_{1^{st}}\underbrace{+1}_{3^{rd}}$ BEDMAS

$(-3,-6)$

$\therefore (-3, 19)$

1^{st}: $|f(x)| \rightarrow$ abs value of y value : $y_1 = -6 \rightarrow y_2 = |-6| = 6$

2^{nd}: $3|f(x)| \rightarrow 3 \times$ y-value : $y_2 = 6 \rightarrow y_3 = 3(6) = 18$

3^{rd}: $3|f(x)|+1 \rightarrow$ up 1 : $y_3 = 18 \rightarrow y_4 = 18 + 1 = 19$

A (20) $y = f(x) \rightarrow$ compressed horizontally (x-values) by $\frac{1}{2}$ (opposite!) $\implies y = f(2x)$

$\rightarrow$ translated 4 units right (opposite!) $\implies y = f(2(x-4))$

$\therefore y = f(2x-8)$

A (21) $y = f(x) + 3$ is translated 3 up from $y = f(x)$

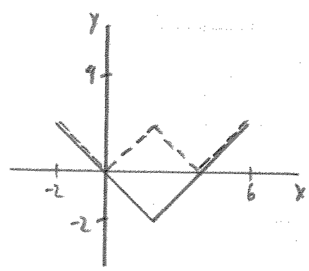
A (22) reflecting in $y=x$ gives inverse f^n : $x = f(y)$ or $y = f^{-1}(x)$

B (23) $y = \sqrt{x} \longrightarrow$ horizontal expanded by factor 3 $\implies y = \sqrt{\frac{1}{3}x}$

(x-values $\rightarrow$ under "$\sqrt{\ }$") (opposite w/ x)

$\longrightarrow$ 2 units right (opposite w/ x, under $\sqrt{\ }$) $\implies y = \sqrt{\frac{1}{3}(x-2)}$

B (24) Reflect negative y-value points in x-axis (i.e. reflect $y = f(x) < 0$ in x-axis):



--- $y = |f(x)|$

— $y = f(x)$

C (25) $y = -f(2x) + 3$

① negate y-value

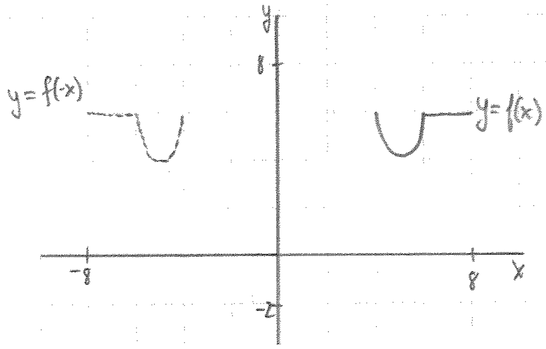
② $\frac{1}{2}$ x-value

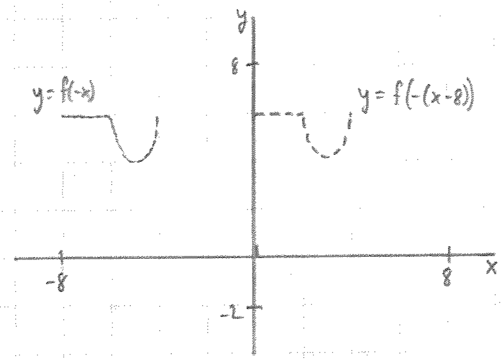
③ add 3 to y-value

$$(8,-6) \xrightarrow{①} (8,6) \xrightarrow{②} (4,6) \xrightarrow{③} (4,9)$$

ORDER: abs value (like brackets), reflections, stretches, translations

B (26) Recall: Reflections THEN Translations





reflect in y-axis to get correct orientation of final image

$\therefore\ y = f(-x)$

translate right 8 units (note scale on graph)

$\therefore\ y = f[-(x-8)]$

B (27) $y = -f(x)$ represents reflection of $y = f(x)$ in x-axis. Note y values negated & x values fixed.

affect y-values

A (28) $y = \frac{2x}{x-1}$ switch $y \leftrightarrow x$: $x = \frac{2y}{y-1}$

$x \neq 1$

$x(y-1) = 2y$ (multiply both sides by $(y-1)$)

$xy - x = 2y$ (expand LS)

$xy - 2y = x$ (subtract $2y$ & add x to both sides)

$y(x-2) = x$ (factor LS)

$y = \frac{x}{x-2}$, $x \neq 2$ (divide both sides by $x-2$)

Restrictions will not be on the test ☺ ... not part of curriculum, sadly ☹

A (29) Notice the x-values have been doubled (y-values fixed), so...

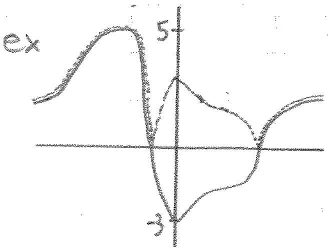
$y = f\left(\frac{1}{2}x\right)$ b/c x is opposite (i.e. $\div 2$ or $\times \frac{1}{2}$) & transformation on x are in $f(\ )$

A (30) See #4 please for explanations,
BUT note that c) & d) are clearly wrong for ANY reciprocal fn b/c the graph passes the x-axis!?!
AND b) is false b/c graph is all positive (for y) when that isn't the case for $y = f(x)$
Isn't it fun to analyze! This is why reciprocal fns are my fav ☺

C ㉛ $y = f(x)$ range is $-3 \le y \le 5$

so $y = |f(x)|$ has range $0 \le y \le 5$

ex



note $0 \le y \le 5$

— $y = f(x)$
---- $y = |f(x)|$

A ㉜ $y = \frac{1}{3} f(2(x-1))$

vertical stretch of factor $\frac{1}{3}$ — horizontal stretch of factor $\frac{1}{2}$ — right 1

$\underbrace{\qquad\qquad}_{\text{multiplicative Trnfs 1st (any order here)}}$

$y = f(x) \to y = \frac{1}{3}f(x) \longrightarrow y = \frac{1}{3}f(2x) \to y = \frac{1}{3}f(2(x-1))$

$(-2, 6) \to (-2, \frac{1}{3}(6)) = (-2, 2) \to (\frac{1}{2}(-2), 2) = (-1, 2) \to (-1+1, 2) = (0, 2)$

OR you can try in 1 step ☺: $y = f(x) \longrightarrow y = \frac{1}{3}f(2(x-1))$

$(-2, 6) \longrightarrow (\frac{1}{2}(-2)+1, \frac{1}{3}(6)) = (0, 2)$

D ㉝ $y = (x+1)^3$

switch y & x: $x = (y+1)^3$

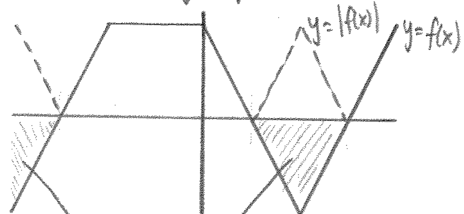
$\sqrt[3]{x} = y + 1$

$\sqrt[3]{x} - 1 = y$

$\therefore$ $y = \sqrt[3]{x} - 1$ or simply! $x = (y+1)^3$ an option! how simple ☺

A ㉞ if -2 & 3 are zeros of $y = f(x)$, then $x = -2$ & $x = 3$ are the vertical asymptotes of $y = \frac{1}{f(x)}$ since the fn is undefined at ~~x=~~ $-2, 3$

C ㉟ given $y = f(x)$, reflect all points for which $f(x) < 0$ in x-axis to get graph of $y = |f(x)|$



negative areas — flip!

C ㊱ $y = -f(2(x+2)) - 3$

negate y-values, $\frac{1}{2}$ x-values — left 2, down 3

$\underbrace{\qquad}_{1^{st}}$ $\underbrace{\qquad}_{2^{nd}}$

$y = f(x) \xrightarrow{1^{st}} y = -f(2x) \xrightarrow{2^{nd}} y = -f(2(x+2)) - 3$

$(6, -5) \longrightarrow (\frac{1}{2}(6), -(-5)) = (3, 5) \longrightarrow (3-2, 5-3) = (1, 2)$

B ㊲ Careful! The wording is poorly chosen here.

$y_1 = f(x) \longrightarrow y_2 = 4f(x-2)$

y_1 vertically expanded by factor $\underline{4}$ — then translated 2 units right

"from the graph y_2" — silly — implies getting y_1 from y_2 but that's not how the question reads