Trig Hwk Booklet Full Solns

for questions covered in Trig Eqns Unit

#6-9, 14-16, 18, 22, 26-29, 35-37, 40, 43, 46-48, 50
52, 55-57, 60, 62, 65, 67, 69 + all write the questions

D ⑥ $\frac{\sin 2\theta}{\sin\theta} = \frac{2\sin\theta\cos\theta}{\sin\theta} = 2\cos\theta$

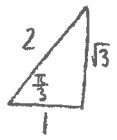
C ⑦ $\frac{\cos x + \cot x}{\sin x + 1} = \frac{\frac{\sin x}{\sin x}\cdot\frac{\cos x}{1} + \frac{\cos x}{\sin x}}{\sin x + 1} = \frac{\frac{\sin x\cos x + \cos x}{\sin x}}{\sin x + 1} = \frac{\cos x(\sin x + 1)}{\sin x}\times\frac{1}{(\sin x + 1)} = \cot x$

B ⑧ $\sin\left(x+\frac{\pi}{3}\right) + \sin\left(x-\frac{\pi}{3}\right) = \sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3} + \sin x\cos\frac{\pi}{3} - \cos x\sin\frac{\pi}{3}$

$= 2\sin x\cos\frac{\pi}{3}$

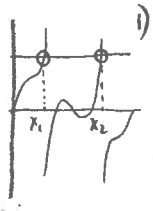
$= 2\sin x\left(\frac{1}{2}\right)$

$= \sin x$



$\cos\frac{\pi}{3} = \frac{1}{2}$

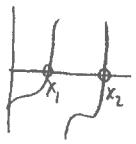
A ⑨ $\sin 3x + \tan x = 3$, $0 \le x \le 2\pi$



i) Do $Y_1 = \sin 3x + \tan x$
$Y_2 = 3$
w/ intersect function
$x \doteq 1.31, 4.31$

OR

ii) $Y_1 = \sin 3x + \tan x - 3$
w/ zero function



$x \doteq 1.31, 4.31$

appropriate window fn:

i) $0 \le x \le 2\pi$ $0 \le y \le 6$
ii) $0 \le x \le 2\pi$ $-2 \le y \le 2$

necessary — suggested

A ⑭ $\cos x = 2x$, $0 \le x < 2\pi$

i) $Y_1 = \cos x$
$Y_2 = 2x$
w/ intersect function

OR

ii) $Y_1 = \cos x - 2x$
w/ zero function

$x \doteq 0.45$

appropriate window:

$0 \le x < 2\pi$ $-4 \le y \le 4$
$\Rightarrow [0, 2\pi, 1]$ by $[-4, 4, 1]$
(Xscl, Yscl)

again, domain more critical than range

D ⑮ $\cos 3x\cos 2x - \sin 3x\sin 2x = \cos(3x + 2x)$

$\alpha = 3x$
$\beta = 2x$
$\cos(\alpha+\beta)$

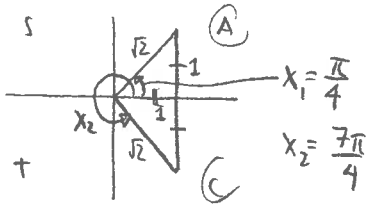
$= \cos(5x)$

C ⑯ $2\cos^2 x - 1 = 0$, $0 \le x < 2\pi$

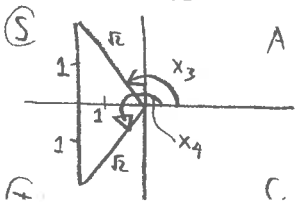
$\Rightarrow \cos^2 x = \frac{1}{2}$

$\Rightarrow \cos x = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$ → $\cos x = \frac{1}{\sqrt{2}}$

memorize your special triangles!



$\cos x = -\frac{1}{\sqrt{2}}$



$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

1 of 1

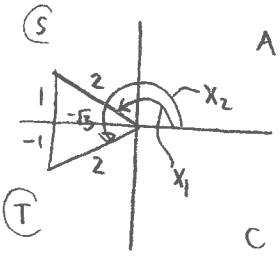
B (18) $\frac{\cos\theta}{\cot\theta} + \frac{1}{\csc\theta} = \cos\theta\tan\theta + \sin\theta = \frac{\cos\theta}{1}\cdot\frac{\sin\theta}{\cos\theta} + \sin\theta = \sin\theta + \sin\theta = 2\sin\theta$

A (22) $\sec\theta\cot\theta\sin\theta = \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1} = 1$

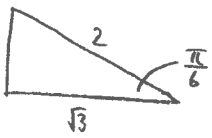
A (26) $2\cos x + \sqrt{3} = 0$, $0 \le x < 2\pi$

$\Rightarrow \cos x = \frac{-\sqrt{3}}{2}$ (S)

Special triangles!



recall



$\therefore X_1 = \pi - \frac{\pi}{6} = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

$X_2 = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

A (27) $\sin 2x + \cos 3x = 1.5$, $0 \le x < 2\pi$ (necessary)

hint: decimal answers? Use graphing calc!!!

$Y_1 = \sin 2x + \cos 3x$

$Y_2 = 1.5$

window: $[0, 2\pi, 1]$ by $[0, 2, 1]$ ← remember that you decide on an appropriate window. This is what I chose bc it fit the intersection pt.

Use Intersect function:

$X = 3.84, 4.37$ (can also use $Y_1 = \sin 2x + \cos 3x - 1.5$ & Zero function)

C (28) $\sin(2x + \pi) = \sin 2x\cos\pi + \cos 2x\sin\pi = \sin 2x(-1) + \cos 2x(0) = -\sin 2x$

$\alpha = 2x$ $\beta = \pi$

D (29) $\sin 3x = 0.4$ has solns $x = 0.14$ & $x = 0.91$

$k = 3 \Rightarrow$ period $= \frac{2\pi}{|k|} = \frac{2\pi}{3}$

So we can add or subtract n multiples of $\frac{2\pi}{3}$ (the period) to each solution to generate more solutions

$\therefore x = 0.14 + \frac{2n\pi}{3}$, n is any integer

$x = 0.91 + \frac{2n\pi}{3}$, n is any integer

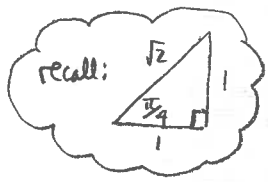
A (35) $\frac{\csc^2 x - 1}{\csc^2 x} = \frac{\cot^2 x}{\csc^2 x} = \left(\frac{\cos x}{\sin x}\right)^2 \cdot \left(\frac{1}{\csc x}\right)^2 = \frac{\cos^2 x}{\sin^2 x}\cdot\frac{\sin^2 x}{1} = \cos^2 x$

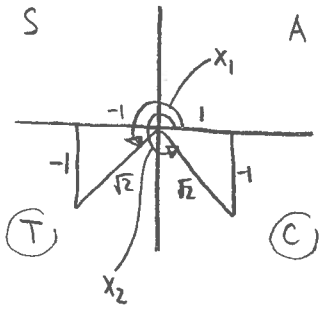
rearrange $1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

D (36) $\sqrt{2}\sin x + 1 = 0$, $0 \le x < 2\pi$

$\Rightarrow \sin x = \frac{-1}{\sqrt{2}}$

recall: 



$x_1 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

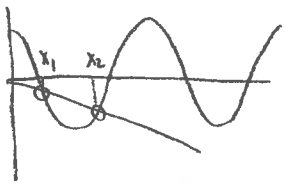
$x_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

D (37) $3\cos 2x = -x$, $0 \le x < 2\pi$

$Y_1 = 3\cos 2x$

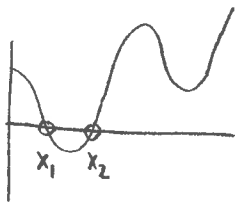
$Y_2 = -x$

use INTERSECT function



or $Y_1 = 3\cos(2x) + x$

w/ ZERO function



Window: $X_{min} = 0$, $X_{max} = 2\pi$ } necessary

$X_{scl} = 1$ (not a big deal)

$Y_{min} = -5$, $Y_{max} = 5$ } amplitude is 3 is a hint

$Y_{scl} = 1$ (not big deal)

$\therefore x_1 \doteq 0.95 \quad x_2 \doteq 1.99$

A (40) $(a\sin x + a)(b\cos x - c) = 0$ for $0 \le x < 2\pi$, $1 < a < b < c$

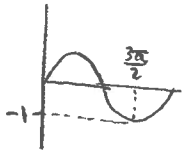
$\Rightarrow a\sin x + a = 0$ or $b\cos x - c = 0$

$\Rightarrow \sin x = \frac{-a}{a} = -1$ or $\cos x = \frac{c}{b} > 1 \because c > b > 1$

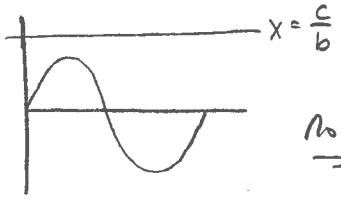
$a \neq 0$ ☺ $\because a > 1$

$b > 1$ & $b \neq 0$

$\Rightarrow$ 1 solution ($x = \frac{3\pi}{2}$ actually) on $0 \le x < 2\pi$



$\Rightarrow$ No solution



No intersection!

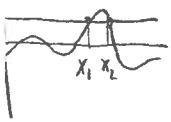
∴ only 1 solution — What a great question for a test!! ☺

C (43) $\sin 2x - \cos x = 1$, $0 \le x < 2\pi$

$Y_1 = \sin 2x - \cos x$

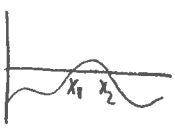
$Y_2 = 1$

use Intersect fn



or $Y_1 = \sin 2x - \cos x - 1$

w/ Zero fn



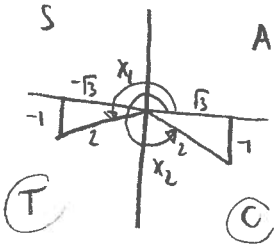
$X_1 = 3.14$

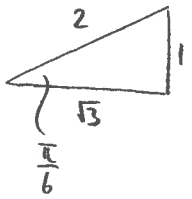
$X_2 = 4.36$

3 of 4

C (46) $2\sin x + 1 = 0$, $0 \le x < 2\pi$

$\Rightarrow \sin x = \frac{-1}{2}$





$X_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$X_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

B (47) $\dfrac{\tan\theta \csc^2\theta}{\sec^2\theta} = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\sin^2\theta} \cdot \dfrac{\cos^2\theta}{1} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

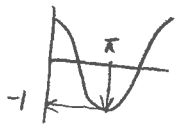
A (48) $\cos(\pi - 2x) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\alpha = \pi$ $\quad = \cos\pi\cos 2x + \sin\pi\sin 2x$

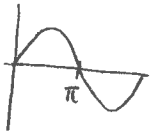
$\beta = 2x$ $\quad \cos(\alpha - \beta) = (-1)\cos 2x + (0)\sin 2x$

$= -\cos 2x$

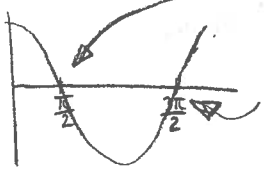
$\cos\pi = -1$



$\sin\pi = 0$



(50) Solution is $\frac{\pi}{2} + n\pi$, $\frac{\pi}{6} + 2n\pi$, $\frac{11\pi}{6} + 2n\pi$



n=1 gives $\frac{\pi}{2} + \pi = \frac{\pi}{2} + \frac{2\pi}{2} = \frac{3\pi}{2}$

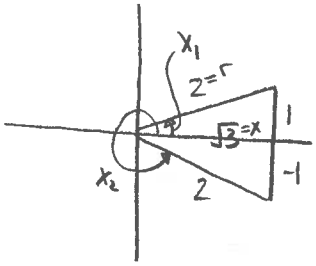
means period is $2\pi \Rightarrow$ of form $\cos x$ (instead of $\cos 2x$, etc)

$X_1 = \frac{\pi}{6}$ + $X_2 = \frac{11\pi}{6}$ are soln's

1st quad

4th quad

Soln is $\cos x = 0$



Special Δ!

So $\cos x = \dfrac{\sqrt{3}}{2}$ ← x, ← r

$\Rightarrow 2\cos x = \sqrt{3}$

$\Rightarrow 2\cos x - \sqrt{3} = 0$

together, we have $\cos x(2\cos x - \sqrt{3}) = 0$

A and C

D

(52) $\dfrac{2\sin\theta}{\sin 2\theta} = \dfrac{2\sin\theta}{2\sin\theta\cdot\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta$

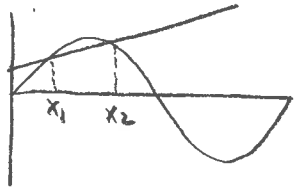
(55) $3\sin x = x+1, \quad 0 \le x < 2\pi$

graphing calc.

$Y_1 = 3\sin x$

$Y_2 = x+1$

use [Intersect] function

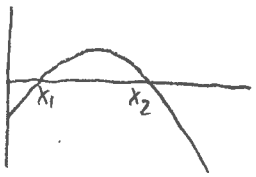


or

amplitude is 3

$Y_1 = 3\sin x - x - 1$

use [zero] function



possible window:

$[0, 2\pi, 1]$ by $[-5, 5, 1]$

$x_1 = 0.54$

$x_2 = 1.87$

D

(56) $\sin\left(\dfrac{3\pi}{2} + x\right)$

$\alpha = \frac{3\pi}{2}$

$\beta = x$

use $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

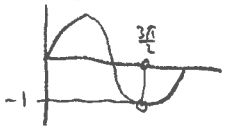
$= \sin\frac{3\pi}{2}\cos x + \cos\frac{2\pi}{2}\sin x$

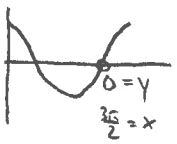
$= (-1)\cos x + (0)\sin x$

$= -\cos x$

graph to check on calc:

$\sin\left(\frac{3\pi}{2}\right) = -1$ $\qquad$ $\cos\left(\frac{3\pi}{2}\right) = 0$





D

(57) $\sin^2 x = \sin x\cos x, \quad 0 \le x < 2\pi$

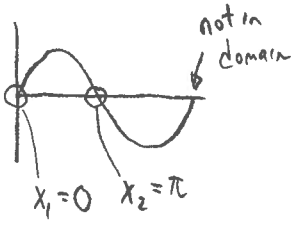
$\Rightarrow \sin^2 x - \sin x\cos x = 0$

$\Rightarrow \sin x(\sin x - \cos x) = 0$

$\Rightarrow \sin x = 0$ or $\sin x - \cos x = 0$

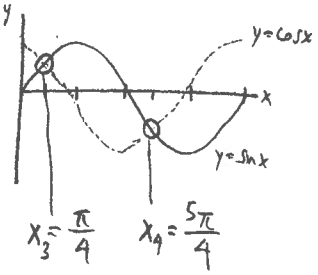
$\therefore x = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

$\sin x = 0$



$\sin x - \cos x = 0$

$\Rightarrow \sin x = \cos x$

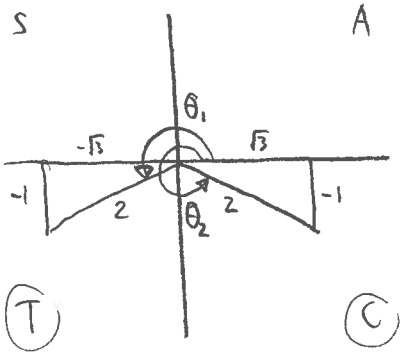


B

(60) $\sin 2x = -\frac{1}{2}$

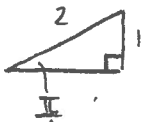
for simplicity, let $\theta = 2x$

then $\sin\theta = -\frac{1}{2}$ ← y, ← r



T $\qquad$ C

recall special Δ:



So $\theta_1 = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\theta_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

But $\theta = 2x$

So $2x_1 = \frac{7\pi}{6}$ or $x_1 = \frac{7\pi}{12}$

$2x_2 = \frac{11\pi}{6}$ or $x_2 = \frac{11\pi}{12}$

In general,

$x = \frac{7\pi}{12} + n\pi$

$x = \frac{11\pi}{12} + n\pi$

} n is any integer

recall $\sin 2x$ has period π:

$k = 2$

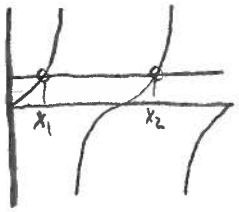
period $= \frac{2\pi}{|k|} = \frac{2\pi}{2} = \pi$

A (62) $\tan x + \sin x = 1$, $0 \le x < 2\pi$

use $Y_1 = \tan x + \sin x$

$Y_2 = 1$

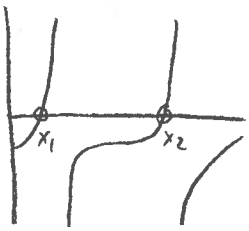
+ intersect function



or

$Y_1 = \tan x + \sin x - 1$

+ zero function



$x_1 = 0.49$

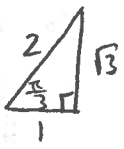
$x_2 = 4.22$

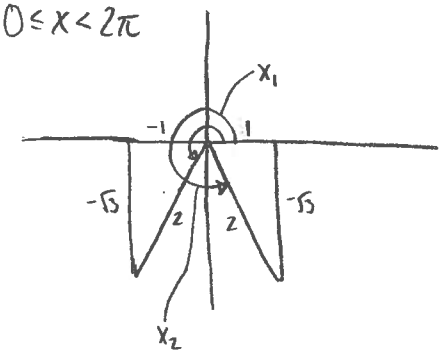
A (65) $\dfrac{\cos 8x + 1}{2} = \dfrac{\cos(2(4x)) + 1}{2} = \dfrac{[2\cos^2(4x) - 1] + 1}{2} = \dfrac{2\cos^2 4x}{2} = \cos^2 4x$

B (67) $\sqrt{3} + 2\sin x = 0$, $0 \le x < 2\pi$

$\therefore 2\sin x = -\sqrt{3}$

$\sin x = \dfrac{-\sqrt{3}}{2} = \dfrac{y}{r}$





$x_1 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x_2 = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

A (69) $3\sin 5x = 1$

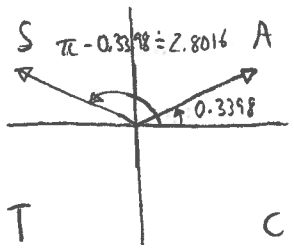
$\sin 5x = \frac{1}{3}$

let $\theta = 5x$

then $\sin\theta = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\doteq 0.3398$



$\theta_1 = 0.3398$

$\theta_2 = 2.8016$

} but $5x = \theta$ $\therefore$

$x_1 \doteq \frac{0.3398}{5} \doteq 0.07$

$x_2 \doteq \frac{2.8016}{5} = 0.56$

In general:

$X = 0.07 + \frac{2n\pi}{5}$

$X = 0.56 + \frac{2n\pi}{5}$

n is any integer

$\sin 5x = \frac{1}{3}$

$\downarrow$

$k = 5$

period is $\frac{2\pi}{5}$

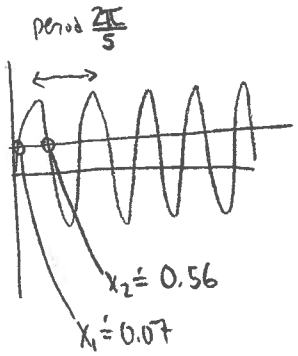
OR

Simply use graphing calc.

$Y_1 = 3\sin 5x$

$Y_2 = 1$

use Intersect function



$x_1 = 0.07 + \frac{2n\pi}{5}$

$x_2 = 0.56 + \frac{2n\pi}{5}$

period

$n \in \mathbb{Z}$