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Trig Hwk Booklet Full Sol^n

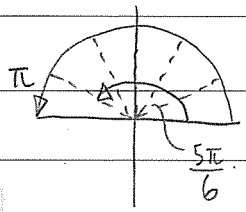
#1-5, 10-13, 17, 19-21, 23-25, 30-34, 38, 39, 41, 42, 44, 45, 49, 51, 53, 54, 58, 59, 61, 63, 64, 66, 68 ... challenge + preview of Math 11 Conc: 70

D ① $\frac{5.3 \text{ rad}}{1} \times \frac{180^\circ}{\pi} \doteq 303.67^\circ$

C ② $y = 6\cos\left(\frac{2\pi}{15}x\right) + 8$

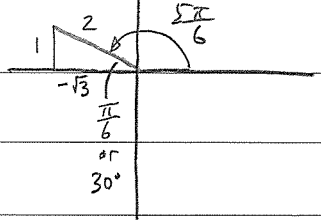
k value period $= \frac{2\pi}{|k|} = \frac{2\pi}{\left(\frac{2\pi}{15}\right)} = \frac{2\pi \cdot 15}{2\pi} = 15$

B ③ $\tan\left(\frac{5\pi}{6}\right) = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$ use ratio of special triangle:

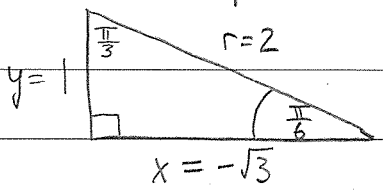


← easy way to "see" $\frac{5\pi}{6}$

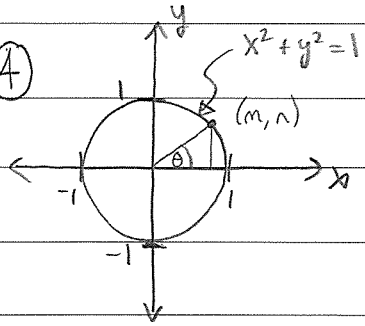
So...



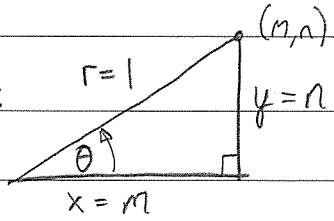
close up:



B ④



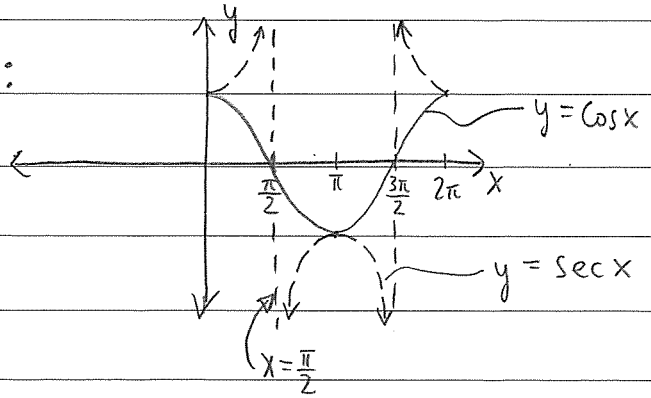
close up:



$\sin\theta = \frac{y}{r} = \frac{n}{1} = n$

C ⑤ $y = \sec x = \frac{1}{\cos x}$ So $\cos x \neq 0 \Rightarrow x \neq \frac{\pi}{2}$ ∴ $x = \frac{\pi}{2}$ is an asymptote

Don't believe it? Graph it:



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see # 20 also

A (10) radius, $r = 18$m looking for eqn of form $h(t) = A \sin Bt + D$

$\Rightarrow$ So $A = 18$ (amplitude)

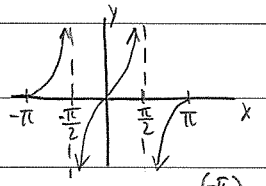
Centre 20m above ground (where ride begins) $h(t) = 18 \sin \frac{\pi}{16} t + 20$

$\Rightarrow D = 20$

1 rotation every 32 s

$\Rightarrow$ takes Ferris wheel 32 s to do 1 cycle of 2π or 2π per 32s $\therefore B = \frac{2\pi}{32} = \frac{\pi}{16}$

C (11) $y = \tan x$ repeats one full cycle every π radians

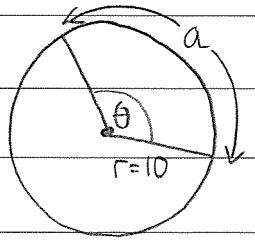


* can use g. calc to graph OR Think $y = \tan x = \frac{\sin x}{\cos x} \Rightarrow x = -\frac{\pi}{2}$: $\frac{\sin\left(-\frac{\pi}{2}\right)}{\cos\left(-\frac{\pi}{2}\right)} = \frac{-1}{0} = -\infty$

$x = 0$: $\frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$x = \frac{\pi}{2}$: $\frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0} = +\infty$

D (12)



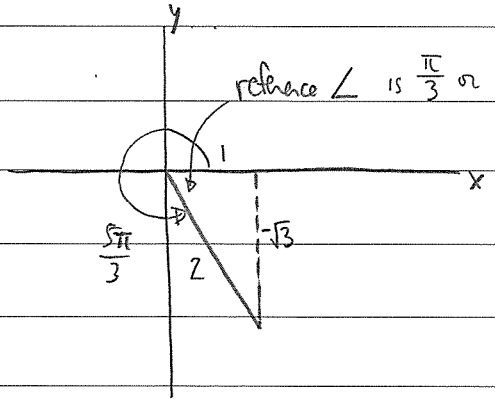
Set up a ratio:

$$\frac{\text{circumference of circle}}{\text{1 full rotation}} = \frac{\text{arc length (part of circle)}}{\text{sector angle (part of rotation)}}$$

$$\frac{2\pi r}{2\pi} = \frac{a}{\theta}$$

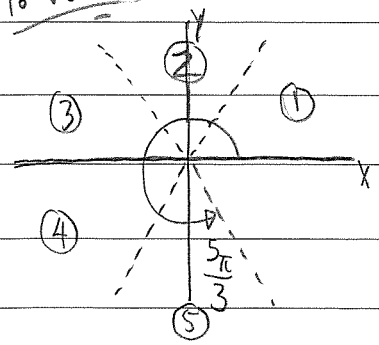
$$\therefore a = \frac{2\pi r \theta}{2\pi} = r\theta = (10\text{ cm})(2) = 20\text{ cm}$$

A (13) (see # 3) $\tan\left(\frac{5\pi}{3}\right)$



$$\therefore \tan\left(\frac{5\pi}{3}\right) = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

To visualize:



Note: 2π split up into thirds of π or $\frac{\pi}{2}$ pieces

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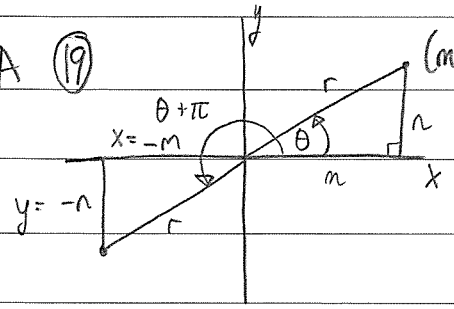
C (17) $f(x) = a\cos x + d$, $a>0$, $d>0$ Max is amplitude + vertical displacement

amplitude — vertical displacement so: $a+d$ (What is the min?)

A (19)



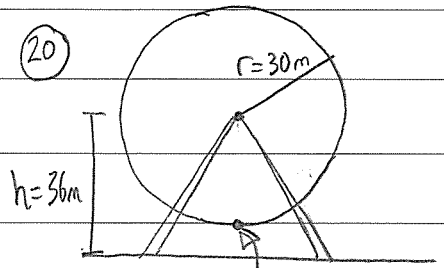
(m, n) must be in 1st quadrant b/c $m>0$, $n>0$

$m^2 + n^2 = r^2$

$\therefore r = \sqrt{m^2+n^2}$ by Pythagorean thm.

$\sin(\pi+\theta) = \frac{y}{r} = \frac{-n}{\sqrt{m^2+n^2}}$

D (20)



begin here ($t=0$) b/c says ride starts at a min

rotates once every 12 seconds ⇒ period = 12 s

$\Rightarrow$ period $= \frac{2\pi}{B} \Rightarrow 12 = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$

Centre 36 m above ground $\Rightarrow D = 36$

amplitude is the radius (30 m) BUT at $t=0$, the ride is at a min & $y=\cos x$ is a max

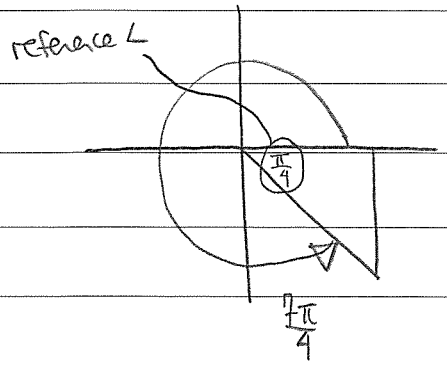
at $x=0 \Rightarrow y = -\cos x$ (gives a min @ $x=0$)

eqn of form: $h(t) = A\cos Bt + D$

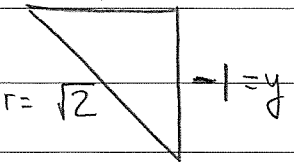
$h(t) = -30\cos\frac{\pi}{6}t + 36$ fits the information $\therefore A = -30$

D (21) $\frac{210^\circ}{1} \times \frac{\pi}{180^\circ} = \frac{7\pi}{6} \doteq 3.67$

D (23) $\sec\frac{7\pi}{4} = \frac{1}{\cos\frac{7\pi}{4}}$ or $\frac{r}{x}$



Close up:



$\therefore \cos\frac{7\pi}{4} = \frac{x}{r} = \frac{1}{\sqrt{2}}$ reciprocal is $\frac{r}{x}$ or $\frac{\sqrt{2}}{1}$

So $\sec\frac{7\pi}{4} = \frac{r}{x} = \sqrt{2}$

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A (24) $y = 3\cos 4x$ period $= \frac{2\pi}{|k|} = \frac{2\pi}{4} = \frac{\pi}{2}$

k or $B = 4$

We used in Unit #1 / book uses

D (25) $y = -2\sin 3x + 4$

amplitude is $|A| = |-2| = 2$

vertical displacement

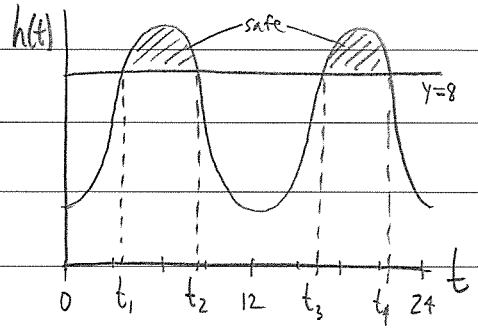
Max (y value): Vertical displacement + amplitude $= 4 + 2 = 6$

Min (y value): V. disc. − amp $= 4 - 2 = 2$

$\therefore 2 \le y \le 6$ is the range

D (30) $h(t) = 3.9\sin 0.16\pi(t-3) + 6.5$

Need to find for what $t \in [0, 24]$ meaning $0 \le t \le 24$ will $h(t) \ge 8$.



Graph it & use the Intersection function with:

$Y_1 = 3.9\sin(0.16\pi(t-3)) + 6.5$

and $Y_2 = 8$

$t_1 = 3.785$ $t_2 = 8.465$ $t_3 = 16.285$ $t_4 = 20.965$

So # hours $= (t_4 - t_3) + (t_2 - t_1)$ or simply $2(t_2 - t_1)$ — why?!

$= 9.36$

D (31) $y = -5\sin\pi(x-3) + 4$

amplitude is $|-5| = 5$

C (32) $\frac{135°}{1} \times \frac{\pi}{180°} = \frac{3\pi}{4} \doteq 2.36$

A (33) $y = \tan 4x$ period of $y = \tan x$ is π (see question 11)

$k = 4$ means graph of $y = \tan x$ is compressed horizontally by factor $\frac{1}{4}$ $\therefore$ period is $\frac{\pi}{4}$ and

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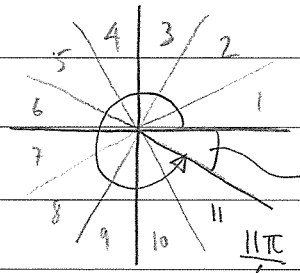
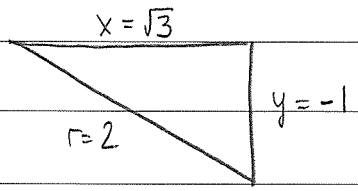
D (34) $\sec \frac{11\pi}{6}$



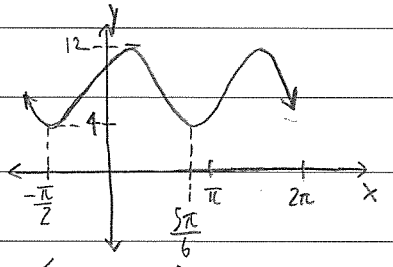
Close up:



$\sec \frac{11\pi}{6} = \frac{r}{x} = \frac{2}{\sqrt{3}}$

- $\frac{1}{2}$ rotation is π
- split in $\frac{\pi}{6}$ sections — need 11 $\frac{\pi}{6}$ pieces!

D (38)

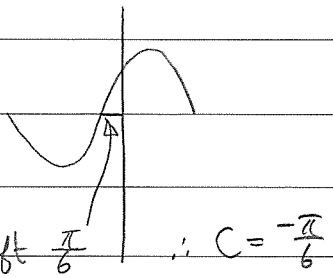


Amplitude $= \frac{|\max - \min|}{2} = \frac{|12-4|}{2} = 4 \Rightarrow A = 4$

Centre or vertical displacement is $\min + \text{amp} = 4 + 4 = 8$ $\quad (\max - \text{amp}) \quad (12-4)$ $\therefore D = 8$

period is $\frac{5\pi}{6} - \left(-\frac{\pi}{2}\right) = \frac{5\pi}{6} + \frac{3\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3} \Rightarrow \frac{4\pi}{3} = \frac{2\pi}{B} \Rightarrow B = \frac{3}{2}$

So for $y = A \sin B(x-C) + D \Rightarrow y = 4 \sin \frac{3}{2}(x-C) + 8$



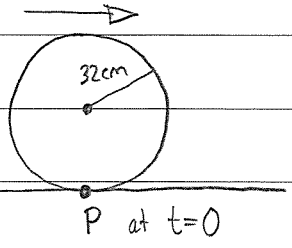
$\therefore C = \frac{-\pi}{6}$

But for $C=0$ & $x=0$, $y=8$

But here for $x = \frac{-\pi}{6}$, $y=8$ $\quad \therefore C = \frac{-\pi}{6}$

$\therefore y = 4 \sin \frac{3}{2}\left(x + \frac{\pi}{6}\right) + 8$

C (39)



(see question 10.20) let $h(t) = A \cos Bt + D$

$= -32 \cos \frac{\pi}{4} t + 32$

h(t) is height of pt P as wheel moves over time t seconds

Radius is 32 cm gives amplitude: 32

Centre is 32 cm above ground: $D = 32$

b/c @ $t=0$, P is a min, then $A<0$ ie $A = -32$

1 rotation in 8 s ie period is 8 s

$\text{period} = \frac{2\pi}{|B|} \Rightarrow 8 = \frac{2\pi}{B}$

$\therefore B = \frac{\pi}{4}$

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D (41) $\frac{5\pi}{2} \times \frac{180^\circ}{\pi} = 450^\circ$

C (42) $y = 4\cos x - 2$

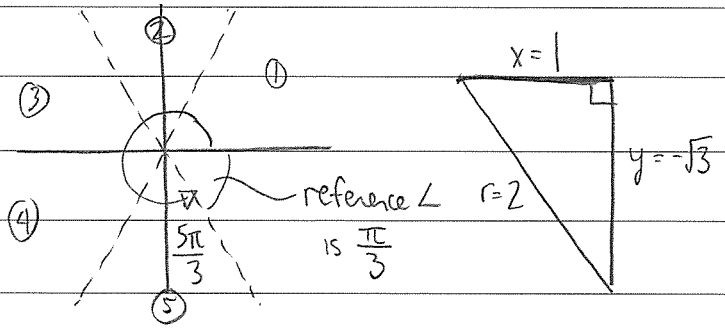
amplitude $|4| = 4$; vertical disc. is down 2

Find: min (y value): v. disc − amp $= -2 - 4 = -6$

max (y value): v. disc + amp $= -2 + 4 = 2$

so range is $-6 \le y \le 2$.

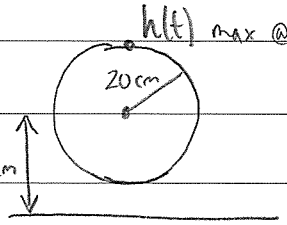
D (44) $\cot \frac{5\pi}{3} = \frac{x}{y} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$



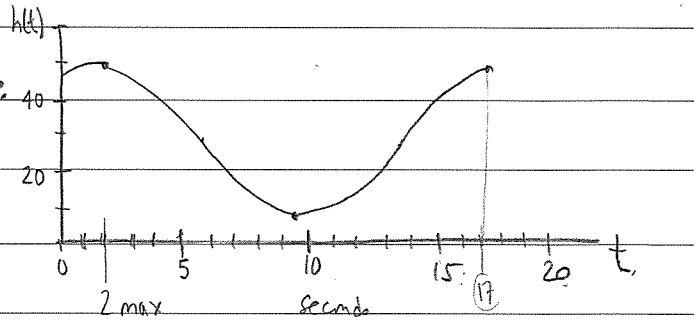
D (45) $f(x) = -\frac{1}{2}\sin\frac{x}{3} = -\frac{1}{2}\sin\left(\frac{1}{3}\right)x$

k or $B = \frac{1}{3}$ ∴ period $= \frac{2\pi}{|B|} = \frac{2\pi}{\left(\frac{1}{3}\right)} = 2\pi \cdot 3 = 6\pi$

B (49)



Can graph it to see: (not necessary)



rotates every 15 s ⇒ period is 15 s ie $B = \frac{2\pi}{15}$

Centre @ 20 cm ⇒ amplitude is 20 cm ie $A = 20$

v. displ. is 30 cm ⇒ $D = 30$ cm

The phase shift is 2 to the right since $h(t) = \cos t$ has max @ $t = 0$ & here max is @ $t = 2$ ⇒ $C = 2$

∴ $h(t) = A\cos B(t - C) + D = 20\cos\frac{2\pi}{15}(t - 2) + 30$

if $t = 2$ is max, then $t = 2 + \frac{15}{2} = 9.5$ is min

& halfway pt is $\frac{9.5 + 2}{2} = 5.75$

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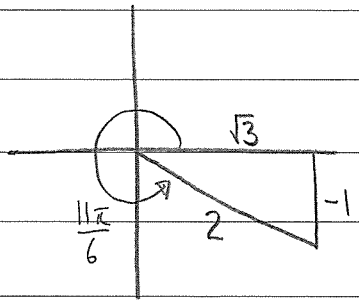
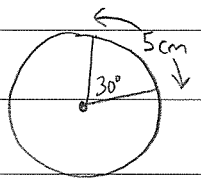
D (51) $\cos\frac{11\pi}{6} = \frac{x}{r} = \frac{\sqrt{3}}{2}$



C (53)



Set up ratio to compare:

$\frac{\text{Circumference}}{1 \text{ rotation}} = \frac{\text{arc length}}{\text{sector angle}}$

(note: $30° = \frac{\pi}{6}$ rad)

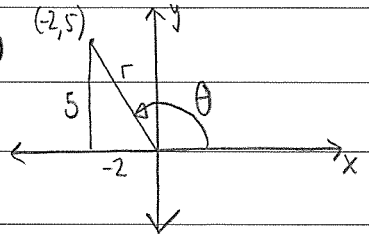
$\frac{2\pi r}{2\pi} = \frac{5\text{cm}}{\pi/6}$

$r = \frac{5 \times 6}{\pi} \doteq 9.55\text{cm}$

$k = \pi$ ... period is $\frac{\pi}{k}$ for $\tan x$ — compare w/ $\sin x$ + $\cos x$ → they're different! } 2π

A (54) $y = \tan \pi x$ period of $y = \tan x$ is π, so period of $y = \tan \pi x$ is $\frac{\pi}{\pi}$ or 1

C (58)



$r = \sqrt{5^2 + (-2)^2}$

$= \sqrt{29}$

$\sec\theta = \frac{r}{x}$ (since $\cos\theta = \frac{x}{r}$ + $\sec\theta = \frac{1}{\cos\theta}$)

$= \frac{\sqrt{29}}{-2}$

$= -\frac{\sqrt{29}}{2}$

B (59) $y = b\cos ax - 2b$, $a>0$, $b>0$

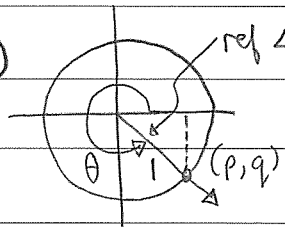
Min value of y: V. disc − amp = $-2b + b = -3b$

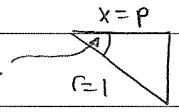
$\therefore$ range is $-3b \le y \le -b$

Max value of y: V. disc + amp = $-2b + b = -b$

C (61) $\frac{150°}{1} \times \frac{\pi}{180°} = \frac{5\pi}{6}$

A (63)





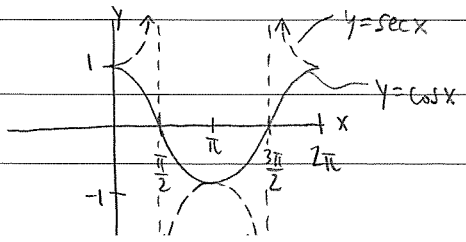
(note $q<0$)

$\cos\theta = \frac{x}{r} = \frac{p}{1} = p$

A (64) $y = \tan\left(\frac{\pi}{5}x\right)$

$k = \frac{\pi}{5}$ period for $y = \tan kx$ is $\frac{\pi}{k} \Rightarrow$ period is $\frac{\pi}{\frac{\pi}{5}} = 5$

A (66) $\sec x = \frac{1}{\cos x}$



Zeroes for $y = \cos x$: $x = \frac{\pi}{2}$ & $x = \frac{3\pi}{2}$ are asymptotes for $y = \sec x$

& $\cos x > 0 \iff \sec x > 0$

$y = \pm 1$ fixed pts

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B (68) $y = a \sin(x-c) + d$, $a, c, d > 0$

amplitude $|2a| = 2a$; vertical displacement d up

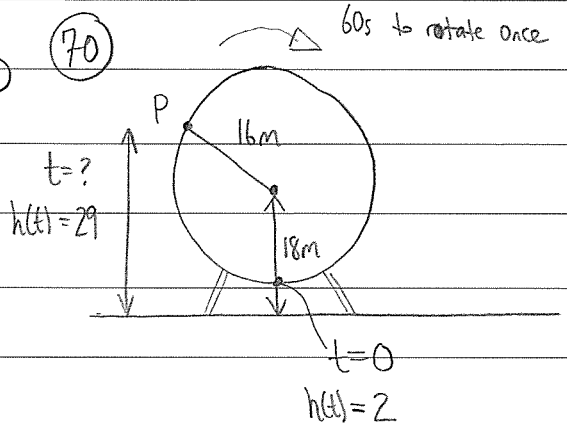
double a for new function: $y = (2a)\sin(x-c) + d$

min y value: V. displacement − amp $= d - 2a$

max y value: V. disc. + amp $= d + 2a$

$\therefore$ range of y is: $d - 2a \le y \le d + 2a$

B (70)



let $h(t) = A\cos Bt + D$ model the height (m) of the Ferris Wheel after t seconds.

Ride begins @ min $\Rightarrow A<0$ + radius is 16m $\Rightarrow A=-16$

Centre is 18m above ground (height) $\Rightarrow D = 18$

Period is 60s $\Rightarrow B = \frac{360°}{60} = 6$

$\therefore h(t) = -16\cos 6t + 18$ (in degrees)

you will learn to do this in the next unit ☺

set $29 = -16\cos 6t + 18$

$\frac{11}{-16} = \cos 6t$ (let $\theta = 6t$ for ease of calculation)

$\therefore \cos\theta = \frac{-11}{16}$

$\therefore \theta = \cos^{-1}\left(\frac{-11}{16}\right)$

$\doteq 2.3288$

but $\theta = 6t \doteq 133.4325$

$\therefore t \doteq 133.4325 \div 6$

$\doteq 22.24$ s